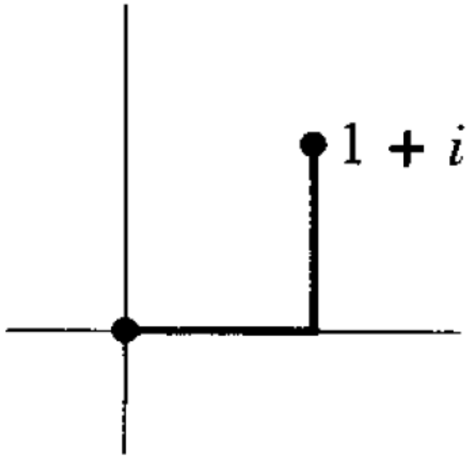
$\int_0^{1+i} (z^2 - z)\, dz$

(a) along the line $y = x$;

(b) along the indicated broken line.



a) To evaluate the integral $\int_0^{1+i} (z^2 - z)dz$ along the line $y = x$ certain conditions are to be followed:
When $y = x$ then $z = x + iy = x(1 + i)$ and $dz = dx + idy = dx(1 + i)$ as $dy = dx$.

$z^2 - z = 2ix^2 - x(1+i)$ Note that integration limits for z changes to the limits of x from $x = 0$ to $x = 1$ (Complex variable limits to →real variable limits)

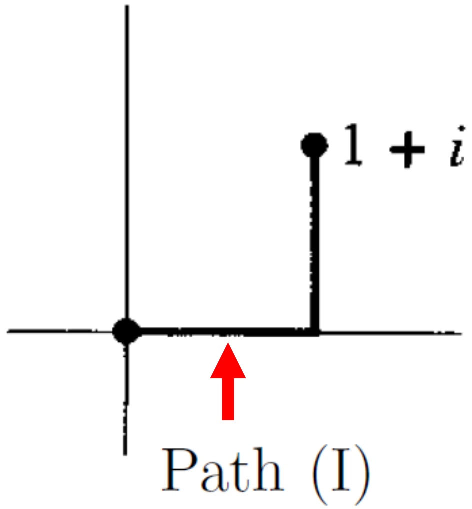
$$\int_0^{1+i} (z^2 - z)dz = \int_{x=0}^{x=1} \left(2ix^2 - x(1+i)\right)(1+i)dx = -\frac{1}{3}(2+i)$$

$$\int_0^{1+i}(z^2 - z)dz = \int_{x=0}^{x=1}\left(2ix^2 - x(1+i)\right)(1+i)dx = -\frac{1}{3}(2+i)$$

b) To evaluate the integral $\int_0^{1+i}(z^2 - z)dz$, we divide the total path into two: Path (I) along x-axis and Path (II) along the line parallel to y-axis at $x = 1$.

Along Path (I): $y = 0, dy = 0, \Rightarrow z = x, dz = dx$
$z^2 - z = x^2 - x.$

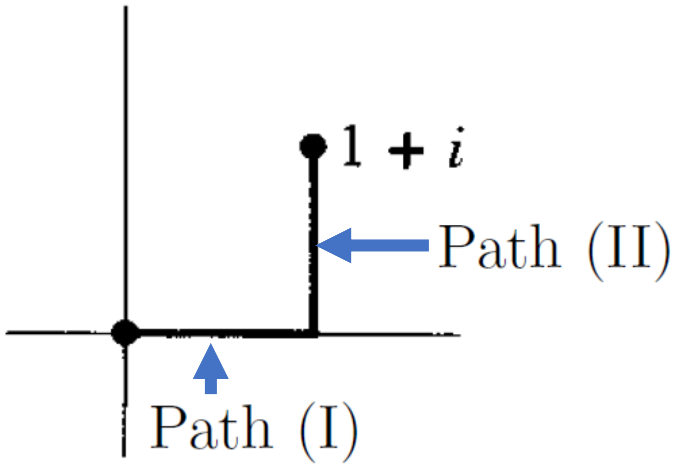
$$\int_0^{1+i}(z^2 - z)dz = \int_{x=0}^{x=1}\left(x^2 - x\right)dx = \frac{1}{3} - \frac{1}{2}$$



Along Path (II): $x = 1, dx = 0, \Rightarrow z = 1 + iy, dz = idy,$

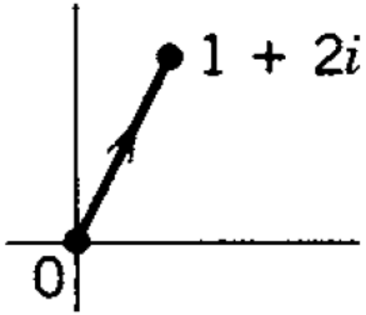
$$\int_0^{1+i} (z^2 - z)dz = \int_{y=0}^{y=1} \left(-y^2 + iy\right) idy = -\frac{i}{3} - \frac{1}{2}$$

$$\int_0^{1+i} (z^2 - z)dz = \int_{Path\ I} (z^2 - z)dz + \int_{Path\ II} (z^2 - z)dz = -\frac{1}{3}(2 + i)$$

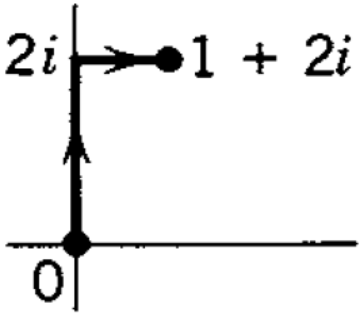


$\int_0^{1+2i} |z|^2\, dz$ along the indicated paths:

(a)



(b)



$\int e^{-z}$ along the positive part of the line $y = \pi$

↓

$$\int_{i\pi}^{\infty+i\pi} e^{-z}\, dz$$

Use Cauchy's theorem or integral formula to evaluate :

$$\oint_C \frac{\sin z \, dz}{2z - \pi} \text{ where } C \text{ is the circle } \begin{array}{l} \text{(a) } |z| = 1, \\ \text{(b) } |z| = 2. \end{array}$$

Use Cauchy's theorem or integral formula to evaluate :

$$\oint_C \frac{\sin 2z \, dz}{6z - \pi} \text{ where } C \text{ is the circle } |z| = 3.$$

Differentiate Cauchy's formula

$$f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)\,dz}{(z-a)^2}$$

By differentiating n times, obtain

$$f^{(n)}(a) = \frac{n!}{2\pi i} \oint_C \frac{f(z)\,dz}{(z-a)^{n+1}}$$

$$\oint_C \frac{\sin 2z \, dz}{(6z - \pi)^3} \text{ where } C \text{ is the circle } |z| = 3$$

$$\oint_C \frac{e^{3z}\,dz}{(z-\ln 2)^4}$$

if C is the square with vertices $\pm 1 \pm i$.