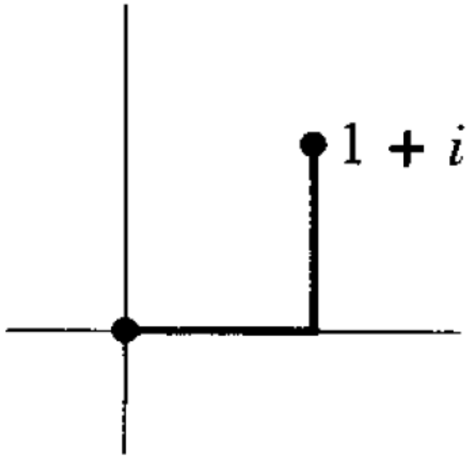
$\int_0^{1+i} (z^2 - z)\, dz$

(a) along the line $y = x$;

(b) along the indicated broken line.



a) To evaluate the integral $\int_0^{1+i}(z^2-z)dz$ along the line $y=x$ certain conditions are to be followed:
When $y=x$ then $z=x+iy=x(1+i)$ and $dz=dx+idy=dx(1+i)$ as $dy=dx$.

$z^2-z=2ix^2-x(1+i)$ Note that integration limits for z changes to the limits of x from $x=0$ to $x=1$ (Complex variable limits to →real variable limits)

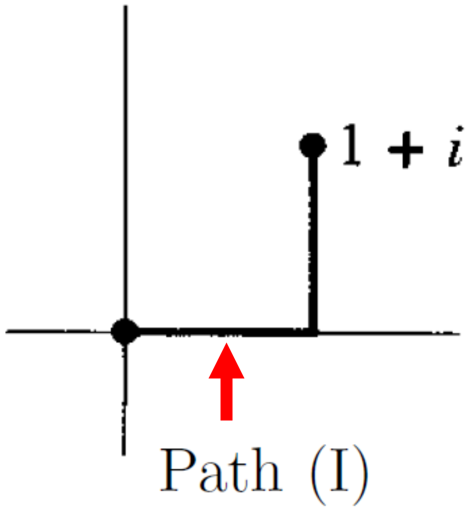
$$\int_0^{1+i}(z^2-z)dz=\int_{x=0}^{x=1}\left(2ix^2-x(1+i)\right)(1+i)dx=-\frac{1}{3}(2+i)$$

$$\int_0^{1+i}(z^2-z)dz=\int_{x=0}^{x=1}\left(2ix^2-x(1+i)\right)(1+i)dx=-\frac{1}{3}(2+i)$$

b) To evaluate the integral $\int_0^{1+i}(z^2-z)dz$, we divide the total path into two: Path (I) along x-axis and Path (II) along the line parallel to y-axis at $x=1$.

Along Path (I): $y=0, dy=0, \Rightarrow z=x, dz=dx$
$z^2-z=x^2-x.$

$$\int_0^{1+i}(z^2-z)dz=\int_{x=0}^{x=1}\left(x^2-x\right)dx=\frac{1}{3}-\frac{1}{2}$$



Along Path (II): $x = 1, dx = 0, \Rightarrow z = 1 + iy, dz = idy,$

$$\int_0^{1+i} (z^2 - z)dz = \int_{y=0}^{y=1} \left(-y^2 + iy\right) idy = -\frac{i}{3} - \frac{1}{2}$$

$$\int_0^{1+i} (z^2 - z)dz = \int_{Path\ I} (z^2 - z)dz + \int_{Path\ II} (z^2 - z)dz = -\frac{1}{3}(2 + i)$$