PHY-H-CC-T-03: ELECTRICY AND MAGNETISM

LECTURE-5 (Pabitra Halder, Assistant Professor, Department of Physics, Berhampore Girls' College)

NETWORK THEORY

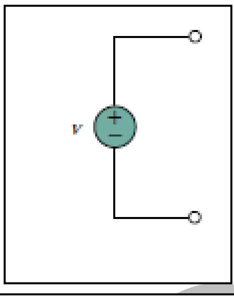
Basic Concepts:

Electric circuit:
Electrical circuit is a interconnection of electrical elements. There are two types of elements found in electric circuits.

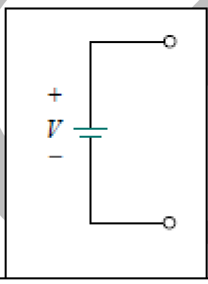
(1) Passive elements:
Passive elements are those which are not capable of generate energy.
Example: resistors, capacitors, and inductors.

(2) Active elements:
Active elements are those which are capable of generate energy.
Examples: generators, batteries, and operational amplifiers.
The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources:

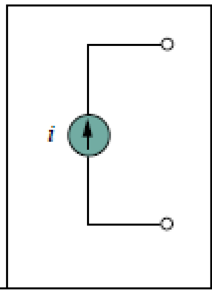
(1) Independent sources:
An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.



Symbol for constant or time varying independent voltage source



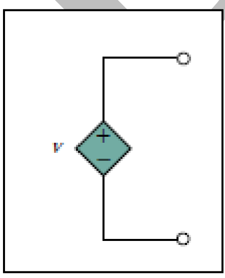
Symbol for constant independent dc voltage source



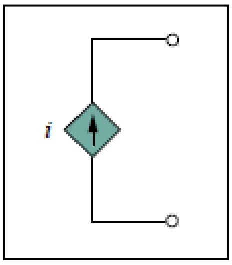
Symbol for independent current source

(2) Dependent sources:

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.



Symbol for dependent voltage source



Symbol for dependent current source

Dependent sources are usually designated by diamond-shaped symbols, as shown in Figure above. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

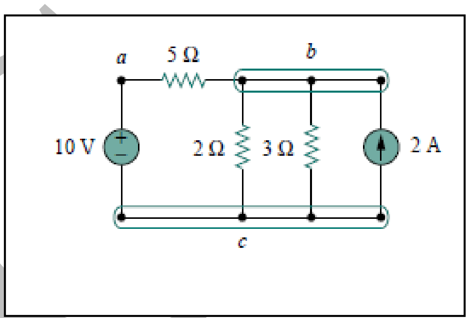
1. A voltage-controlled voltage source (VCVS). 2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS). 4. A current-controlled current source (CCCS).
Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits.

Branches, Nodes, mesh and loops:

Branch:

A branch represents a single element such as a voltage source or a resistor.

The circuit in figure has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.



Node:

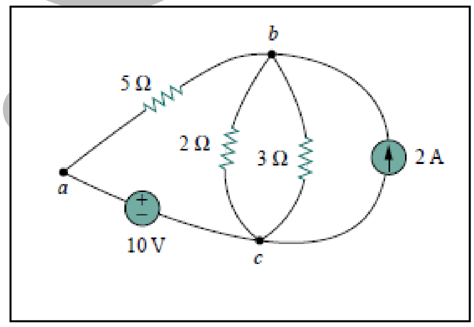
A node is the point of connection between two or more branches.

The circuit in Figure has three nodes a, b, and c.

Loop:

A loop is any closed path in a circuit.

The closed path abca containing two resistors in figure is a loop. Another loop is the closed path bcb containing the 3Ω resistor and the current source. Although one can identify six loops in figure, only three of them are independent.



A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

$$b = l + n - 1$$

Mesh:

A mesh is loop which does not contain any other loops within it.

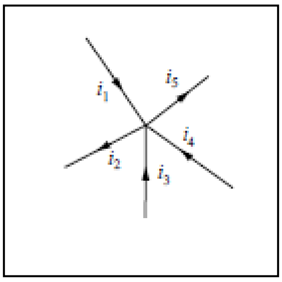
The closed path bcb containing the 3Ω resistor and the current source is a mesh but the closed path abca is not a mesh it is a loop.

Kirchhoff's Laws:

Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as
(1) Kirchhoff's current law (KCL) and
(2) Kirchhoff's voltage law (KVL).

(1) Kirchhoff's current law (KCL):

Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.



Kirchhoff's current law (KCL) states that the algebraic sum of current entering A node (or a closed boundary) is zero.
Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$

Where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

(2) Kirchhoff's voltage law (KVL):

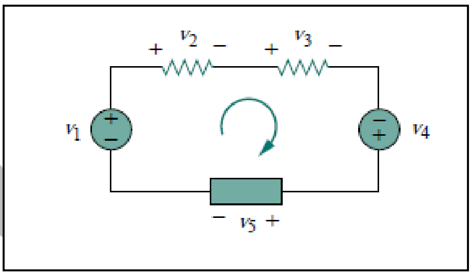
Kirchhoff's second law is based on the principle of conservation of energy. It states that the algebraic sum of all voltages around a closed path (or loop) is zero.
Mathematically, KVL implies that

$$\sum_{m=1}^{M} v_m = 0$$

Where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the m th voltage.

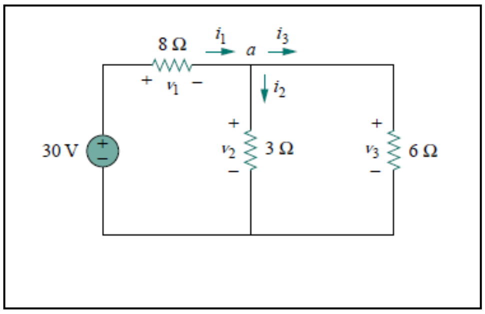
To illustrate KVL, consider the circuit in figure beside. From KVL we can write,

$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$



Example:

Find the current and voltages in the circuit shown in figure beside?



Ans. We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$v_1 = 8i_1$, $v_2 = 3i_2$, $v_3 = 6i_3$ ·······(1)

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v1,v2,v3) or (i1,i2,i3).

At node a, KCL gives $i_1 - i_2 - i_3 = 0$ ········ (2)

Applying KVL to loop 1 as in figure,

$-30 + v_1 + v_2 = 0$

$-30 + 8i_1 + 3i_2 = 0$ (Using equation (1))

$i_1 = \frac{30-3i_2}{8}$ ············ (3)

Applying KVL to loop 2,

$-v_2 + v_3 = 0 \rightarrow v_3 = v_2$ ············ (4)

Using equation (1) we get,

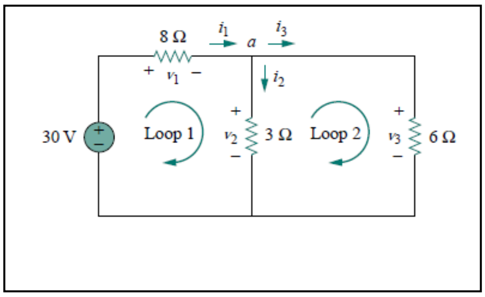
$6i_3 = 3i_2 \rightarrow i_3 = \frac{i_2}{2}$ ············ (5)

Substituting equation (3) and (5) into (2) gives,

$\frac{30-3i_2}{8} - i_2 - \frac{i_2}{2} = 0 \rightarrow \frac{30-3i_2-8i_2-4i_2}{8} = 0 \rightarrow \frac{30-15i_2}{8} = 0 \rightarrow i_2 = 2$ A

From the value of i_2, we now use equation (1) to (5) to obtain

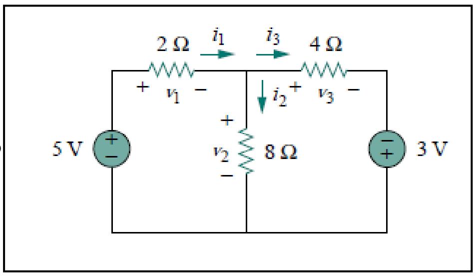
$i_1 = 3$ A, $i_3 = 1$ A, $v_1 = 24$ V, $v_2 = 6$ V and $v_3 = 6$ V



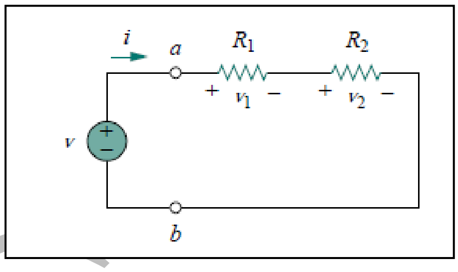
Problem:

Find the currents and voltages in the circuit shown in figure beside?

Answer: v1 = 3 V,v2 = 2 V,v3 = 5 V,i1 = 1.5 A,i2 = 0.25 A, i3 =1.25 A.



Series resistors and voltage division:



The two resistors are in series, since the same current *i* flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$v_1 = i\,R_1$, $v_1 = i\,R_2$ ·········· (1)

If we apply KVL to the loop (moving in the clockwise direction), we have

$v - v_1 + v_2 = 0$ ··········· (2)

Combining equations (1) and (2), we get

$v = v_1 + v_2 = i\,(R_1 + R_2) \rightarrow i = \frac{v}{R_1 + R_2}$ ············· (3)

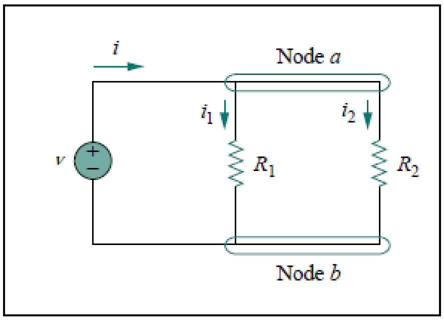
To determine the voltage across each resistor, we substitute equation (3) into equation (1) and obtain

$$v_1 = \frac{R_1}{R_1 + R_2}\,v, \qquad v_2 = \frac{R_2}{R_1 + R_2}\,v$$

The source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the **principle of voltage division** and the circuit in figure is called a **voltage divider**. In general, if a voltage divider has N resistors (R1, R2, ..., RN) in series with the source voltage v, the nth resistor (Rn) will have a voltage drop of

$$v_1 = \frac{R_n}{R_1 + R_2 + \cdots + R_N}\,v$$

Parallel resistors and current division:



Consider the circuit in figure, where two resistors are connected in parallel and therefore have the same voltage across them.

From Ohm's law,

$v = i_1\,R_1 = i_2\,R_2 \rightarrow i_1 = \frac{v}{R_1}$ and $i_2 = \frac{v}{R_2}$ ··········· (1)

Applying KCL at node a gives the total current *i* as

$i = i_1 + i_2$ ············· (2)

Substituting equation (1) into equation (2), we get

$i = \frac{v}{R_1} + \frac{v}{R_2} = v\,(\frac{1}{R_1} + \frac{1}{R_2}) = \frac{v}{R_{eq}} \rightarrow v = i\,R_{eq}$ ·············· (3), (where, $R_{eq} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$)

Combining equation (1) and (3) results in

$$i_1 = \frac{R_2}{R_1 + R_2}\,i, \qquad i_2 = \frac{R_1}{R_1 + R_2}\,i$$

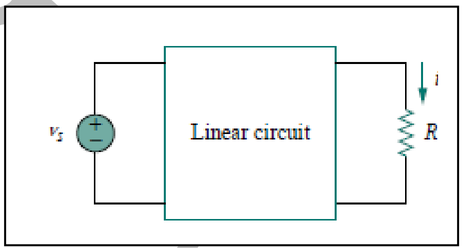
The total current *i* is shared by the resistors in inverse proportion to their resistances. This is known as **the principle of current division**, and the circuit in figure is known as a **current divider**. Notice that the larger current flows through the smaller resistance.

Circuit theorems:

A major disadvantage of analyzing circuits using Kirchhoff's laws is that, for a large, complex circuit, tedious computation is involved. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include Thevenin's and Norton's theorems. Since these theorems are applicable to linear circuits, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of superposition, source transformation, and maximum power transfer.

Linear circuit:

A linear circuit is one whose output is linearly related (or directly proportional) to its output.



To understand the linearity principle, consider the linear circuit shown in figure. The linear circuit has no independent sources inside it. It is excited by a voltage source v_s, which serves as the input. The circuit is terminated by a load R. We may take the current i through R as the output. Suppose v_s = 10 V gives i = 2 A. According to the linearity principle, v_s = 1 V will give i = 0.2 A. By the same token, i = 1 mA must be due to v_s = 5 mV.

Superposition principle:

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.

2. Dependent sources are left intact because they are controlled by circuit variables.

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. Repeat step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

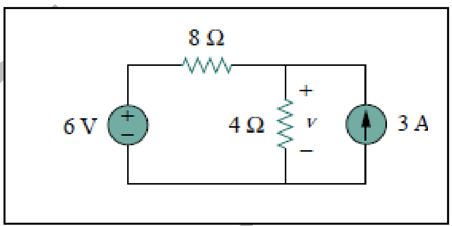
Disadvantage:

Analyzing a circuit using superposition has one major disadvantage:

it may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source. However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits. Keep in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

<u>Examples:</u>

1.Use the superposition theorem to find *v* in the circuit in figure.



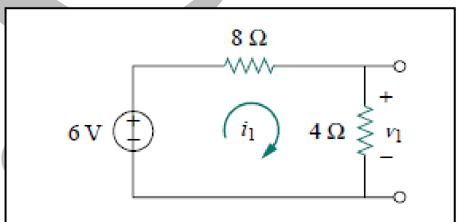
Ans. Since there are two sources, let

$v = v_1 + v_2$

Where v_1 and v_2 are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain v_1, we set the current source to zero, as shown in figure (a). Applying KVL to the loop in figure (a) gives

$12i_1 - 6 = 0 \rightarrow i_1 = 0.5$ A

Thus $v_1 = 4\, i_1 = 2$ V



(a)

(We may also use voltage division to get v_1 by writing

$v_1 = \frac{4}{4+8} \times 6 \text{ V} = 2 \text{ V})$

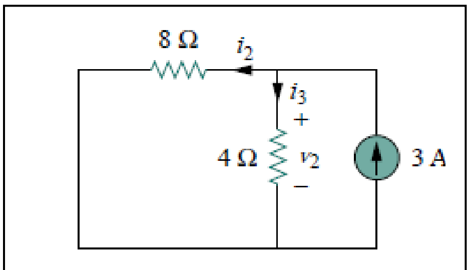
To get v_2, we set the voltage source to zero, as in figure (b).

Using current division,

$i_3 = \frac{8}{4+8} \times 3 \text{ A} = 2 \text{ A}$

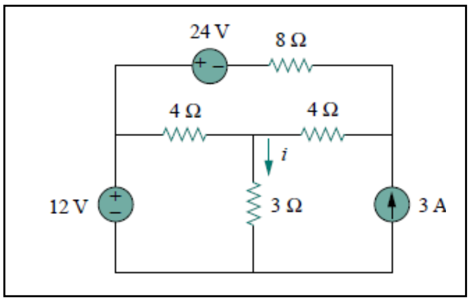
Hence, $v_2 = 4\, i_3 = 4 \times 2 \text{ V} = 8 \text{ V}$

From superposition principle, $v = v_1 + v_2 = (2+8) \text{ V} = 10 \text{ V}$



(b)

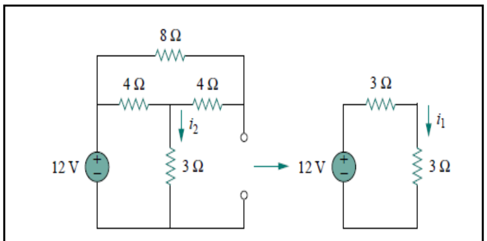
2. For the circuit in figure beside, use the superposition theorem to find *i*.



Ans. In this case, we have three sources. Let

$i = i_1 + i_2 + i_3$

where i_1, i_2,and i_3 are due to the12 V,24 V, and 3A sources respectively. To get i_1, consider the circuit in figure (a).



(a)

Combining 4 Ω (on the right-hand side) in series with 8Ω gives 12Ω.

The 12Ω in parallel with 4Ω(on the left-hand side)gives 12×4/16 = 3Ω.

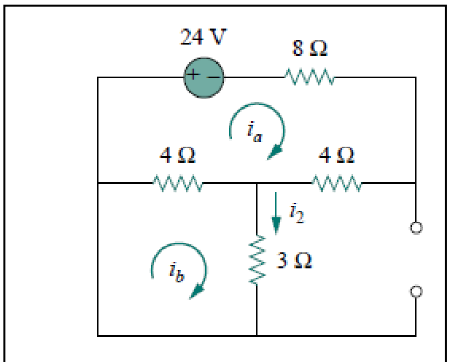
Thus,

$i_1 = \frac{12}{3+3} = 2$ A

To get i_2 , consider the circuit in figure (b). Applying mesh analysis,

$8\ i_a + 4\ i_a + 4\ (i_a - i_b) + 24 = 0 \rightarrow 4i_a - i_b = -6$ ………. (1)

And, $4\ (i_b - i_a) + 3\ i_b = 0 \rightarrow 7\ i_b - 4i_a = 0 \rightarrow i_a = \frac{7}{4}\ i_b$ ……… (2)



(b)

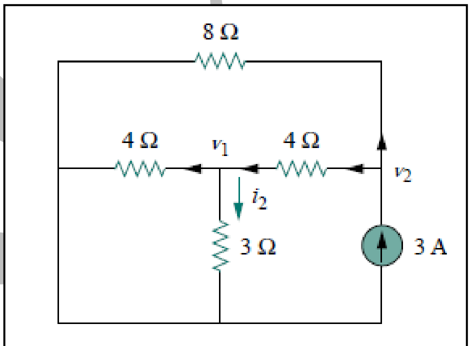
Substituting equation (2) into equation (1) gives

$i_2 = i_b = -1$ A

To get i_3 , consider the circuit in figure (c). Using nodal analysis,

$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \rightarrow 24 = 3\ v_2 - 2\ v_1$ ………… (3)

And , $\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \rightarrow v_2 = \frac{10}{3}\ v_1$ ………… (4)



(c)

Substituting equation (4) into equation (3) leads to

$v_1 = 3$ V and $i_3 = \frac{v_1}{3} = 1$ A

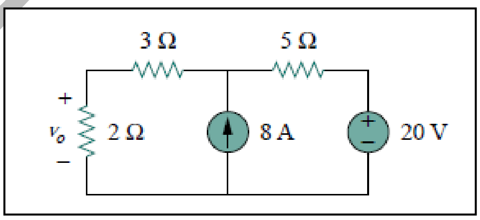
Thus from superposition principle,

$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2$ A

Problem:

1. Using the superposition theorem, find v_0 in the circuit in figure below.

Ans. 12 V



2. Find *i* in the circuit in figure below using the superposition principle.

Ans. 0.75 A