1

SUPERPOSITION OF SIMPLE HARMONIC OSCILLATIONS

1.1 Superposition of simple harmonic motions

If the same motion occurs repeatedly after a definite interval of time, it is called periodic motion. Simple harmonic motion is the *simplest* kind of periodic motion.

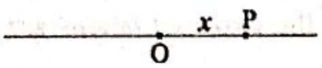


Fig. 1.1

When a particle P, Fig.1.1, has a simple harmonic motion, we know that its distance (x) from the mean position (O) at time t is given by

$$x = a\sin(\omega t + \delta)$$

Here a is amplitude, ω is angular frequency of oscillation and δ is *initial* phase (epoch) of the motion. Initial phase means phase at $t = 0$ (when we start our observation or description).

Now we like to study superposition of two or more simple harmonic motions, i.e., we like to see what happens when a particle has *more than one* simple harmonic motion *at the same time*.

We know that when a particle has two velocities, it moves with the resultant velocity, but each component velocity produces its own effect. Similarly, when a particle has two simple harmonic motions at the same time, each produces its own effect. The resultant motion of the particle may be of various types, depending upon the relation between two superposed motions.

The resultant motion can be determined by using the principle of superposition.

Principle of superposition **:** When two or more simple harmonic motions superpose, the resultant displacement at any instant is the *sum* of individual displacements at that instant. If the displacements at an instant owing to two simple harmonic motions are x_1 and x_2, then the resultant displacement at that instant is given by

$$x = x_1 + x_2 \quad \text{.....................1.1}$$

It simply means that *each individual motion produces its own effect* and the resultant motion is the sum of the two motions.

1.1.1 *Linearity and Principle of Superposition*

Principle of superposition, eqn. 1.1, is a consequence of the fact that the basic equation of S.H.M. is a *linear* differential equation:

$$\frac{d^2x}{dt^2} = -\omega^2 x \quad \text{..................1.2}$$

It is a linear equation, because the dependent variable x and its derivative have power 1.

It can be shown easily that if x_1 and x_2 are two solutions of this equation, then any *linear combination* of x_1 and x_2 is also a solution. A linear combination of x_1 and x_2 can be written as

$$x = c_1x_1 + c_2x_2 \quad \text{................1.1a}$$

where c_1 and c_2 are constants.

Differentiating both sides of eqn.1.1a, we get

$$\frac{d^2x}{dt^2} = c_1\frac{d^2x_1}{dt^2} + c_2\frac{d^2x_2}{dt^2} = -\,\omega^2(c_1x_1 + c_2x_2) = -\omega_2 x$$

Thus x is also solution of eqn.1.2. Eqn. 1.1 is a special case of eqn.1.1a. Hence the above principle of superposition is a consequence of *linearity* of the equation of S.H.M.

1.2 Superposition of two collinear simple harmonic oscillations

Now we shall study the effects of the superposition of two simple harmonic motions along the *same line* having different amplitudes, phases and frequencies.

(1) Superposition of two simple harmonic motions having the *same frequency and phase* but *different amplitudes* along the *same line*.

We suppose that two forces act on a particle producing two simple harmonic motions along the X-axis. The two motions are represented by the equations:

$x_1 = a_1 \sin(\omega t+\delta)$ and $x_2 = a_2 \sin(\omega t + \delta)$ 1.3

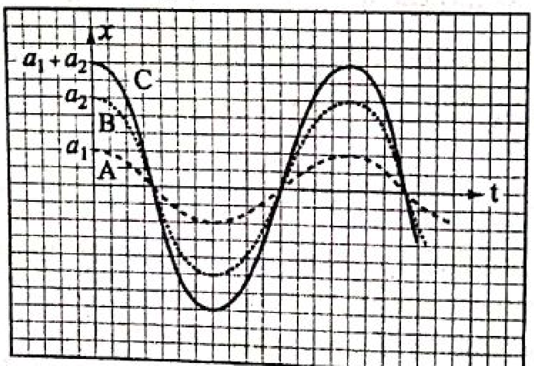
The two motions have the same frequency ω, the same phase $(\omega t + \delta)$ but different amplitudes a_1 and a_2.

According to the principle of superposition, the resultant displacement at any instant is given by

$$x = x_1 + x_2 = a_1 \sin(\omega t+\delta) + a_2 \sin(\omega t+\delta)$$

$$\therefore x = (a_1+a_2) \sin(\omega t + \delta) \text{ ....1.4}$$

We see that the resultant vibration is *simple harmonic* and has the frequency and phase *unchanged*, but the amplitude is just the *sum* of the individual amplitudes.



Same phase

Fig. 1.2

In Fig. 1.2, you may see how the *x-t* curve for the resultant oscillation can be drawn by drawing *x-t* curves for the superposing oscillations and adding the individual displacements. As the phases of the two are the same, displacements at any instant are in the *same* direction so these are added.

(2) Superposition of two simple harmonic motions having the *same frequency* but opposite *phase* and *different amplitudes* along the *same line*.

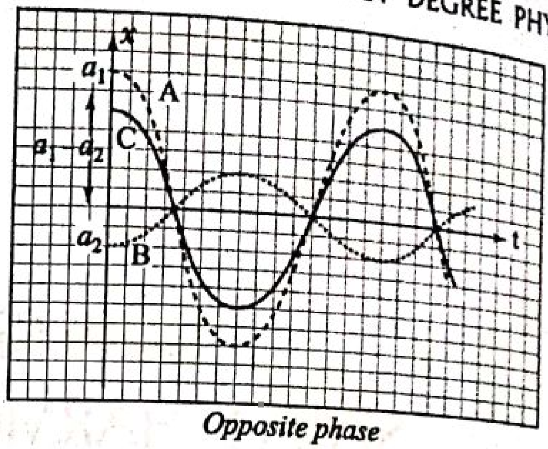
Opposite phase means phase difference is π or odd integral multiple of π.

$$\therefore x_1 = a_1 \sin(\omega t + \delta) \text{ and } x_2 = a_2 \sin(\omega t+\delta+\pi)$$

$$= -a_2 \sin(\omega t + \delta)$$

According to the principle of superposition, the resultant displacement at any instant is given by

$$x = x_1 + x_2 = (a_1 - a_2) \sin(\omega t + \delta)$$



Opposite phase

Fig. 1.3

In Fig. 1.3, you may see how the *x-t* curve for the resultant oscillation can be drawn by drawing *x-t* curves for the superposing oscillations and *subtracting* one displacement from the other, as these are in opposite directions.

(3) Superposition of two simple harmonic motions having the *same frequency* but *different phases and amplitudes* along the *same line*.

Suppose the two oscillations are represented by

$$x_1 = a_1 \sin \omega t \text{ and } x_2 = a_2 \sin(\omega t + \delta).$$

Here δ is the *initial* phase difference between the two superposing oscillations.

According to the principle of superposition, the resultant displacement at any instant is given by

$$x = x_1 + x_2 = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin\omega t + a_2 \sin \omega t \cos\delta + a_2 \cos \omega t \sin \delta$$

$$= (a_1 + a_2 \cos \delta)\sin\omega t + a_2 \sin \delta \cos \omega t \text{ ....(i)}$$

We put $(a_1+a_2\cos \delta) = A \cos\epsilon$ and $a_2 \sin \delta = A \sin\epsilon$ ....(ii)

Then the above eqn. (i) becomes

$$x = A\cos\epsilon \sin\omega t + A\sin\epsilon \cos\omega t = A \sin(\omega t+\epsilon)$$

Hence the particle will oscillate with the *same* frequency ω but with *different* amplitude A and *different* initial phase ϵ. To find the value of A, we square both sides of eqns. (ii) and add. We get

$$A^2(\sin^2\epsilon + \cos^2\epsilon) = (a_1+a_2\cos\delta)^2 + (a_2\sin\delta)^2$$

$$\therefore A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta \text{ ....1.5}$$

Also we find from eqn.(ii)

$$\tan\epsilon = \frac{a_2 \sin\delta}{a_1 + a_2 \cos\delta} \text{ ....1.6}$$

We see that the amplitude and initial phase of the resultant oscillation depend on individual amplitudes a_1 and a_2 and their initial phase difference δ.

Special cases :

(a) The two oscillations have the same phase.

$\therefore$ Initial phase difference, $\delta = 0$.

Resultant amplitude has the maximum value, $A_{max} = a_1 + a_2$.

Initial phase of the resultant oscillation, $\epsilon = 0$.

This situation has been discussed above.

(b) The two oscillations have *opposite* phase.

$\therefore$ Initial phase difference, $\epsilon = \pi$

Resultant amplitude has the minimum value,

$A_{min} = |a_1 - a_2|$.

Initial phase of the resultant oscillation, $\epsilon = 0$.

This situation has also been discussed above.

(c) The two oscillations have *arbitrary* phase relation.

δ has any value other than 0 and π.

In this case eqns.1.5 and 1.6 are the same formula we get when two vectors are added (triangle rule).

Thus we get a very simple rule to add two sinusoidal waves.

We regard the individual amplitudes (a_1 and a_2) as *two vectors* and the phase difference (δ) as the angle between them. Resultant amplitude (A) is equal to the magnitude of the vector sum ($\vec{a}_1 + \vec{a}_2$) and ϵ is the angle the vector makes with the first vector $\vec{a}_1$, Fig1.4.

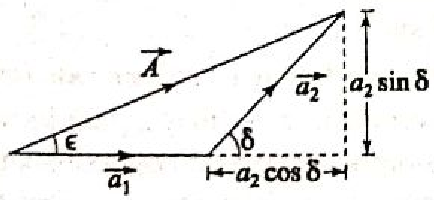


Fig. 1.4

This is a very useful result and can be generalised for any number of simple harmonic motions of the same frequencies superposed on each other. It is the polygon rule.Let us see it in detail.

Suppose that there are four simple harmonic motions :

$y_1 = a_1 \sin(\omega t + \delta_1)$, $y_2 = a_2 \sin(\omega t + \delta_2)$,
$y_3 = a_3 \sin(\omega t + \delta_3)$ and $y_4 = a_4 \sin(\omega t + \delta_4)$.

To add them we consider four vectors $\vec{a}_1, \vec{a}_2, \vec{a}_3$ and $\vec{a}_4$ which make angles δ_1, δ_2, δ_3 and δ_4 respectively with positive X-axis and draw them one after another as shown in Fig. 1.4a.

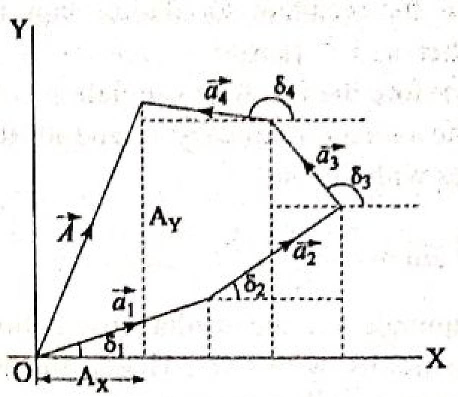


Fig. 1.4a

X and Y-components of the resultant vector $\vec{A}$ are the *algebraic* sum of the corresponding components of these four vectors. From the figure it should be clear why algebraic sum is necessary.

$$A_X = a_1\cos\delta_1 + a_2\cos\delta_2 + a_3\cos\delta_3 + a_4\cos\delta_4$$

and $A_Y = a_1\sin\delta_1 + a_2\sin\delta_2 + a_3\sin\delta_3 + a_4\sin\delta_4$.

Resultant vector has the magnitude and direction given by

$$A = \sqrt{A_X^2 + A_X^2} \text{ and } \tan\delta = \frac{A_Y}{A_X}.$$

Therefore, resultant vibration is given by

$$y = A\sin(\omega t + \delta)$$

(4) Superposition of two simple harmonic motions of *slightly different frequencies* but of the same *initial phase and amplitudes* along the *same line*.

Let the two superposing oscillations be given by $x_1 = a\sin\omega_1 t$ and $x_2 = a \sin \omega_2 t$

The difference between angular frequencies ω_1 and ω_2 is small.

Resultant oscillation is given by

$$x = x_1 + x_2 = a[\sin\omega_1 t + \sin\omega_2 t]$$

$$= 2a \sin\frac{(\omega_1 + \omega_2)}{2} t \cos\frac{(\omega_1 - \omega_2)}{2} t$$

$$= 2a\cos\frac{(\omega_1 - \omega_2)}{2} t \sin\frac{(\omega_1 + \omega_2)}{2} t \quad \text{.............(iii)}$$

We see that x depends on the *product* of two sinusoidal functions of time. One has a large frequency

$\frac{(\omega_1 + \omega_2)}{2} = \omega'$ (average frequency of ω_1 and ω_2)

and the other has a small frequency $\frac{(\omega_1 - \omega_2)}{2} = \omega$.

The frequency ω is small, because ω_1 and ω_2 are very near to each other.

Thus the resultant oscillation, eqn. (iii), can be written as $x = A\sin\omega' t$

Therefore the resultant oscillation is found to have the average frequency ω' and its amplitude A varies with time as

$$A = 2a\cos\frac{(\omega_1 - \omega_2)}{2}t = 2a\cos\omega t$$

Amplitude A of the resultant oscillation *varies* cosinusoidally *with time*. Hence the resultant oscillation is *not* simple harmonic.

Now let us see how A varies with time.

Let $\omega = \dfrac{\omega_1 - \omega_2}{2} = 2\pi\dfrac{(n_1 - n_2)}{2} = 2\pi n$, where n_1 and n_2 are the frequencies of the superposing vibrations and n is *half of* the difference of the two frequencies. Amplitude A varies with frequency n.

Maximum value of the amplitude is $A_{max} = 2a$, it occurs when

$\cos\omega t = \pm 1$, i.e., $2\pi nt = p\pi$, where p is integers 0,1, 2, 3.....

Minimum value of the amplitude is $A_{min} = 0$, it occurs when

$\cos\omega t = 0$, i.e., $2\pi nt = (2p+1)\dfrac{\pi}{2}$.

Hence amplitude varies periodically between A_{max} and A_{min}. This phenomenon is called *beats*.

Maximum amplitude occurs at time

$t = 0, \dfrac{1}{2n}, \dfrac{2}{2n}, \dfrac{3}{2n}$ ....

Minimum amplitude occurs at time

$t = \dfrac{1}{4n}, \dfrac{3}{4n}, \dfrac{5}{4n}$ .....

Time interval between two successive maxima or two successive minima is $\dfrac{1}{2n}$.

One maximum and the next minimum together is called *one beat*.

$\therefore$ One beat is produced in time interval $\dfrac{1}{2n}$ second.

$\therefore$ Number of beats produced in one second is $2n = (n_1 - n_2)$.

Number of beats produced in one second is called beat frequency.

$\therefore$ Beat frequency $= (n_1 - n_2) =$ difference in frequency of the two superposing oscillations.

Beats is a very important phenomenon in sound. The resultant sounds become periodically louder (waxing) and lower (waning). One waxing and the next waning together are called one beat.

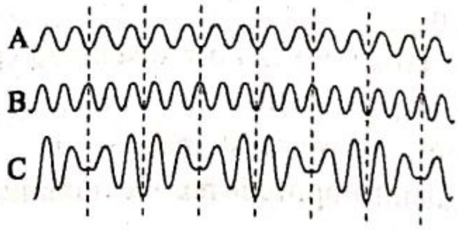


Fig. 1.5

In Fig. 1.5 we see the resultant pattern of vibration C, when two vibrations A and B having frequency ratio 4/5 superpose.

1.3 Superposition of two simple harmonic motions of the *same frequency* but along two lines at *right angle* to each other

(A) Analytical method

We suppose that a particle P has two simple harmonic motions of the same frequency at the same time, one along X-axis and the other along Y-axis. We are to find the resultant motion.

We start with the *general case* when amplitudes are different and the phase difference between them is φ.

$\therefore$ The two motions are represented by the equations:

$x = a\sin\omega t$

$y = b\sin(\omega t + \varphi)$......................1.7

φ = phase difference between two oscillations.

The particle moves along *a path* for which the x and y coordinates of each point satisfy the above two equations *simultaneously*. To find the locus of the particle, we have to eliminate t from the above two equations.

Expanding the second equation we get

$y = b\sin\omega t\cos\varphi + b\cos\omega t\sin\varphi$

Substituting the values of $\sin\omega t$ and $\cos\omega t$ from the first eqn. we get

$$y = b\frac{x}{a}\cos\varphi + b\sqrt{1 - \frac{x^2}{a^2}}\sin\varphi$$

Or, $\left(y - b\dfrac{x}{a}\cos\varphi\right)^2 = b^2\left(1 - \dfrac{x^2}{a^2}\right)\sin^2\varphi$

Or, $y^2 - 2b\frac{x}{a}\cos\varphi\, y + b^2\frac{x^2}{a^2}\cos^2\varphi$

$$= b^2\sin^2\varphi - b^2\frac{x^2}{a^2}\sin^2\varphi$$

Or, $\frac{b^2}{a^2}x^2(\sin^2\varphi + \cos^2\varphi) - \frac{2bxy}{a}\cos\varphi + y^2$

$$= b^2\sin^2\varphi$$

Or, $$\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\varphi + \frac{y^2}{b^2} = \sin^2\varphi \quad \text{........1.8}$$

This is general equation of an ellipse enclosed inscribed in a rectangle *whose sides are* $2a$ *and* $2b$. The major axis of the ellipse makes angle α with the X-axis given by

$$\tan 2\alpha = \frac{2ab}{a^2 - b^2}\cos\varphi \quad \text{..................1.9}$$

Therefore, *in general*, the particle $P(x, y)$ moves along an *elliptical* path, Fig. 1.6. But depending on the phase difference φ and amplitudes the path of the particle may be straight line or circle as we shall see below.

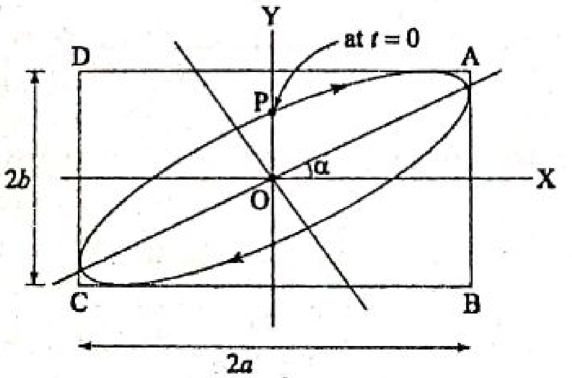


Fig. 1.6

Thus we find that the *periodic motion* of a particle along an elliptical path may be regarded as the result of superposition of two simple harmonic motions of the same frequency at *right* angle to each other.

Now we shall examine interesting *special cases.*

We shall see how the nature of the resultant motion changes in a systematic manner as the phase difference and amplitudes takes different values.

Case I : *Phase difference between the superposing vibrations is* 0 (same phase).

Putting $\varphi = 0$, from eqn.(1.8) we find

$$\left(\frac{y}{b} - \frac{x}{a}\right)^2 = 0$$

$$\therefore y = \frac{b}{a}x \quad \text{.....................1.10}$$

It is equation of a straight line passing through the origin and having a slope

$$\tan\theta = m = \tan^{-1}\frac{b}{a}.$$

Therefore the particle $P(x,y)$ moves *to and fro along the straight line* AC, which is the diagonal of the rectangle, shown in Fig.1.7a.

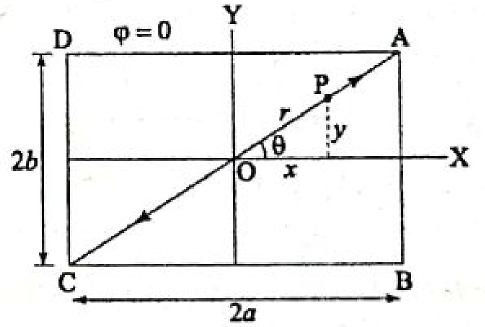


Fig. 1.7a

Distance (r) of the particle P from the origin is given by

$$r^2 = x^2 + y^2$$

$$r^2 = (a^2 + b^2)\sin^2\omega t$$

or, $$r = \sqrt{a^2 + b^2}\sin\omega t$$

Thus the vibration of the particle is simple harmonic having the *same frequency* but amplitude is $\sqrt{a^2 + b^2}$.

Case II : *Phase difference between the superposing vibrations is* $\frac{\pi}{4}$.

Putting $\varphi = \frac{\pi}{4}$ in eqns. 1.8 and 1.9, we get equation of an ellipse, whose major axis makes angle α with the X-axis given by

$$\tan 2\alpha = \frac{2ab}{a^2 - b^2}\frac{1}{\sqrt{2}} \quad \therefore \alpha \text{ is positive.}$$

To find whether the particle moves clockwise or anticlockwise, we have to look at the original vibrations, eqns. 1.7. At $t = 0$, $x = 0$, $y = b\sin\varphi$ =+ve and $\frac{dx}{dt} = a\omega\cos\omega t$ =+ve. Therefore x coordinate of the particle increases from 0 as time increases from 0. Hence the particle moves *clockwise* along an ellipse, as shown in Fig1.6.

Case III : *Phase difference between the superposing vibrations is* $\frac{\pi}{2}$.

Putting $\varphi = \frac{\pi}{2}$, we get from eqns. 1.8 and 1.9 :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ and } \alpha = 0 \quad \text{.................1.11}$$

So, the particle P (x, y) revolves along the ellipse as shown in Fig.1.7b with a period $T=\frac{2\pi}{\omega}$.

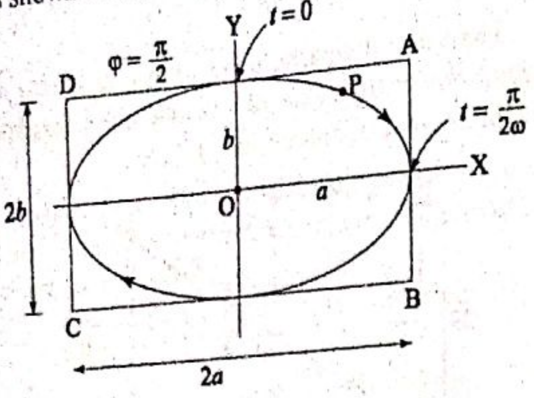


Fig. 1.7b

The major and minor axes *coincide* with the X and Y-axis respectively.

In this case $x = a \sin\omega t$, $y = b \cos \omega t$.

At $t = 0$, $x = 0$, $y = b$. At $t=\frac{\pi}{2\omega}$, $x = a$, $y = 0$, as shown in the figure. Therefore the particle rotates *clockwise.*

Case IV : *Phase difference between the superposing vibrations is* $\frac{\pi}{2}$ *and amplitudes are equal.*

Putting $a=b$ in eqn. 1.11, we get

$$x^2 + y^2 = a^2 \quad \text{..............................1.12}$$

In this case the *ellipse becomes a circle.* The particle P(x, y) revolves *clockwise* along the circle of radius a, Fig. 1.7c. Its time period is $T=2\pi / \omega$, ω is the angular frequency. In this case we can say that ω is the angular velocity of the particle.

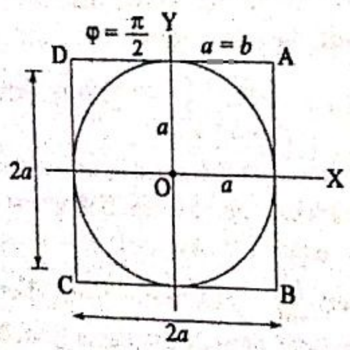


Fig. 1.7c

In the above three cases (II, III, IV) the resultant oscillation is periodic, but *not simple harmonic.*

In case IV we get the relation between a uniform circular motion of radius a and constant angular velocity ω with simple harmonic oscillation.

A uniform circular motion may be thought of as the result of superposition of two simple harmonic motions, of the same frequency and amplitude but with a phase difference $\frac{\pi}{2}$, at right angle to each other given by

$$x = a \sin \omega t \text{ and } y = a \cos \omega t$$

Case V : *Phase difference between the superposing vibrations is* $\varphi=\frac{3\pi}{4}$.

Putting $\varphi=\frac{3\pi}{4}$, we get from eqns. 1.8 and 1.9 equation of an ellipse and the angle α given by

$$\tan 2\alpha = \frac{2ab}{a^2-b^2}\cos\frac{3\pi}{4}$$

Since $\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\text{ve}$. $\therefore$ α is $-$ve. So, major axis of the ellipse makes *negative* angle α with the X-axis, as shown in Fig.1.7d. To find whether the particle moves *clockwise* or anticlockwise, we look at the original vibrations, eqns. 1.7.

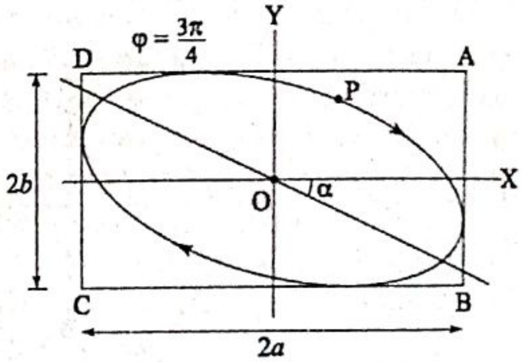


Fig. 1.7d

$$x = \sin\omega t,\ y = b \sin\left(\omega t + \frac{3\pi}{4}\right)$$

At $t = 0$ $x=0$, $y = b \sin\frac{3\pi}{4} = +\text{ve}$, $\frac{dx}{dt} = +\text{ve}$.

Therefore the particle rotates in *clockwise* direction as shown in the figure.

Case VI : *Phase difference between the superposing vibrations is* π (opposite phase).

Putting $\varphi = \pi$, we get from eqns. 1.8 and 1.9:

$$y=-\frac{b}{a}x \quad \text{......................1.13}$$

It is an equation of a straight line passing through the origin and having a slope

$-\tan\theta = m = -\tan^{-1}\frac{b}{a}$. (Slope is *negative*).

Therefore the particle P(x,y) moves along the straight line BD, which is the *other diagonal* DB of the rectangle, shown in Fig, 1.7e. By the same argument as we had in case I, we can find that the particle has the same kind of simple harmonic vibration as in case I.

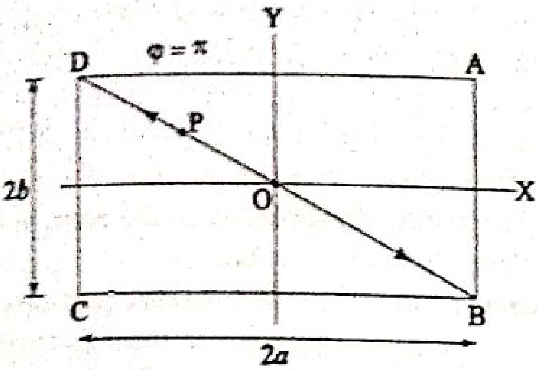


Fig. 1.7e

Case VII : *Phase difference between the superposing vibrations is* $\varphi = \frac{5\pi}{4}$.

Putting $\varphi = \frac{5\pi}{4}$, we get from eqns. 1.8and 1.9, we get an equation of an ellipse and the angle α given by

$$\tan 2\alpha = \frac{2ab}{a^2 - b^2} \cos \frac{5\pi}{4} = -\text{ ve}$$

So, major axis of the ellipse makes *negative* angle α with the X-axis, as shown in Fig.1.7d.

To find whether the particle moves clockwise or anticlockwise, look at the original vibrations, eqns. 1.7.

$$x = \sin \omega t,\ y = b\sin\left(\omega t + \frac{5\pi}{4}\right)$$

At $t = 0$ $x=0$, $y = b \sin\frac{5\pi}{4} = -\text{ve}$, $\frac{dx}{dt} = +\text{ve}$.

Therefore the particle rotates in *anticlockwise* direction *opposite* to what is shown in the figure. Notice, *the direction has changed.*

Case VIII : *Phase difference between the superposing vibrations is* $\varphi = \frac{3\pi}{2}$.

Putting $\varphi = \frac{3\pi}{2}$, we get from eqns. 1.8 and 1.9 :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } \alpha = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots 1.11$$

So, the particle P(x, y) revolves along the ellipse, whose major axis coincides with the X-axis as shown in Fig.1.7c with a period $T = \frac{2\pi}{\omega}$.

$$\text{At } t = 0, x = 0, y = b\sin\left(\omega t + \frac{3\pi}{2}\right) = -b.\ \frac{dx}{dt} = +\text{ve}.$$

Therefore, as time t increases from t=0, x increases. Therefore, the particle rotates *anticlockwise opposite* to what is shown in the figure. The *direction has changed.*

If $\varphi = \frac{3\pi}{2}$ and the two amplitudes are *equal* ($a = b$) the *ellipse becomes* circle, as discussed in case IV. The particle rotates *anticlockwise.*

Case IX : *Phase difference between the superposing vibrations is* $\varphi = \frac{7\pi}{4}$.

Putting $\varphi = \frac{7\pi}{4}$, we get from eqns. 1.8 and 1.9, we get an equation of an ellipse and the angle α given by

$$\tan 2\alpha = \frac{2ab}{a^2 - b^2} \cos \frac{7\pi}{4} = +\text{ ve}$$

So, major axis of the ellipse makes *positive* angle α with the X-axis, as shown in Fig.1.6.

At $t = 0$, $x = 0$, $y = b \sin\frac{7\pi}{4} = -\text{ve}$, $\frac{dx}{dt} = +\text{ ve}$.

Therefore, as time t increases from t=0, x increases. Therefore, the particle rotates *anticlockwise* opposite to what is shown in the figure. The *direction has changed.*

Case X : *Phase difference between the superposing vibrations is* $\varphi = 2\pi$ (same phase)

This situation is identical with case I. So the particle will vibrate along the diagonal AC, as shown in Fig.1.7a.

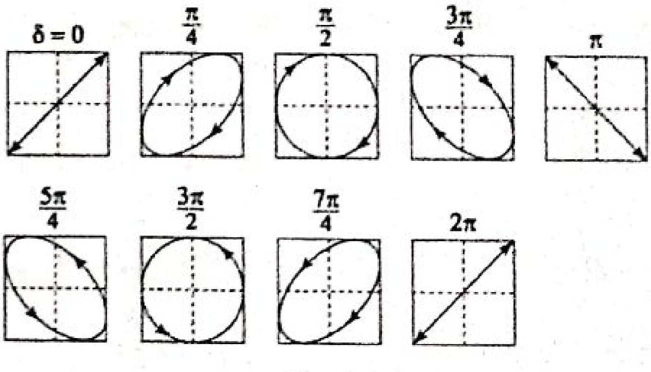


Fig. 1.8

Now we summarise the above results in a figure. In Fig.1.8 we can see the changes in the path along which the particle moves, as the phase difference (φ) between the two superposing simple harmonic motions increases in steps of π /4 from 0 to 2π. We have taken $a = b$. If the phase difference increases further, the pattern of motion changes in the reverse order.

(B) Graphical method

The resultant oscillation produced by

superposition of two simple harmonic oscillations at right angle to each other can also be found by *graphical* method. To apply this method we utilise the basic facts: (i) when a particle moves with a uniform angular velocity ω in a circle of radius a, the foot of the perpendicular from the particle on a diameter moves in a simple harmonic motion of amplitude a and angular frequency ω. The circle and particle are called the *reference circle* and *reference particle*. (ii) if base and height of a right angled triangle are two simultaneous displacements, then the hypotenuse is the resultant displacement.

We shall now describe the graphical method for two phase differences.

The two oscillations are represented by eqns.

$$x = a \sin \omega t \text{ and } y = b \sin (\omega t + \varphi)$$