PHY-H-CC-T-03: ELECTRICY AND MAGNETISM

LECTURE-1 (Pabitra Halder (PH), Department of Physics, Berhampore Girls' College)

Magnetic effect of steady current

1. Lorentz force:

The force exerted on point charge q by an electric field $\vec{E}$ is $\overrightarrow{F_e}$= q$\vec{E}$. But here we assume that the charge q is at rest. If the charge moves with a velocity $\vec{v}$ an addition force appear, this force is called Lorentz force or magnetic force. This force is given by $\overrightarrow{F_m}$= q ($\vec{v} \times \vec{B}$), where $\vec{B}$ is known as the magnetic induction or magnetic field vector. $\overrightarrow{F_m}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. If q is positive the direction of $\overrightarrow{F_m}$ is obtained by the right handed cork-screw rule.

The Lorentz force $\overrightarrow{F_m}$ is perpendicular to $\vec{v}$. Hence, the work-done by $\overrightarrow{F_m}$ on q as q moves through a distance $\overrightarrow{\Delta l}$(=$\vec{v}$ dt) is Δw= $\overrightarrow{F_m}$. $\overrightarrow{\Delta l}$ = $\overrightarrow{F_m}$. $\vec{v}$ dt =0

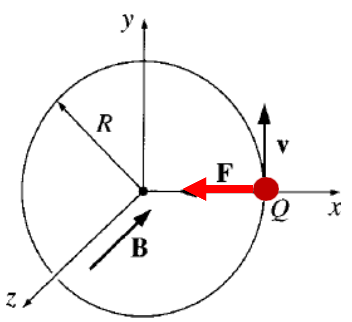
This implies Lorentz force acting alone does no work on the charged particle.

In the presence of both electric and magnetic fields, the net force on q would be $\vec{F}$= q ($\vec{E}$ + $\vec{v} \times \vec{B}$).

Example 1:

Cyclotron motion

If a charge particle of mass m and charge Q moves with a velocity $\vec{v}$ in a region of uniform magnetic field $\vec{B}$ which is perpendicular to the direction of motion of charge particle. Then this magnetic field experience Lorentz force QvB on the charge particle which moves in a circular orbit of definite radius. This archetypical motion of charge particle in a magnetic field is called cyclotron motion. This motion is periodic and the frequency of this motion is called cyclotron frequency.



In equilibrium, Lorentz force supplied by the magnetic field to balance the centripetal force acting on the charge particle Q to sustain the circular motion.

$$QvB=\frac{mv^2}{R}$$

$$mv=QBR$$

Therefore $p=QBR$ > Cyclotron formula

Cyclotron frequency:

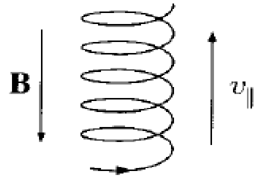
For cyclotron motion we can write

$$m\omega^2 R=QvB$$

$\omega=\frac{QB}{m}$ > Cyclotron frequency

Special case:

If the charge particle start out with some additional speed $v_{\|}$ parallel to $\vec{B}$, this component of motion is unaffected by the magnetic field and this particle moves in helix. The radius is still given by the equation $R=\frac{mv_{\perp}}{QB}$. But this velocity $v_{\perp}$ is now the component perpendicular to $\vec{B}$.



Application:

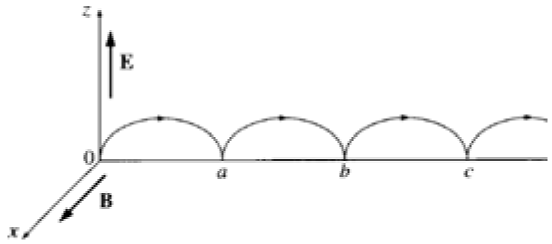
1. The motion of a particle in cyclotron is the first modern particle accelerator.

2. Cyclotron motion of the particle is the simple experimental technique for finding the momentum of the particle: send it through a region of known magnetic field, and measure the radius of its circular trajectory.

Example 2:

Cycloid Motion:

A more exotic trajectory occurs if we include a uniform electric field, at right angle to the magnetic one. Suppose, for instant, that $\vec{B}$ points in x-direction, and $\vec{E}$ in the z-direction as shown in figure. A particle at rest is released from the origin; what path will it follow?



Qualitative discussion:

- Initially, the particle is at rest, so the magnetic force is zero.
- The electric field accelerates the charge in the z-direction.
- As it picks up speed, a magnetic force develops.
- The magnetic field pulls the charge around to the right.
- The faster it goes, the stronger magnetic force becomes; it curves the particle back around towards the *y* axis.
- The charge is moving *against* the electrical force, so it begins to slow down.
- the magnetic force then decreases and the electrical force takes over, bringing the charge to rest at point *a*.
- There the entire process commences anew, carrying the particle over to point *b*, and so on.

Quantitative discussion:

$$\mathbf{v} = (0, \dot{y}, \dot{z}) \longrightarrow \mathbf{v}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 0 & \dot{y} & \dot{z} \\ B & 0 & 0 \end{vmatrix} = B\dot{z}\,\hat{\mathbf{y}} - B\dot{y}\,\hat{\mathbf{z}}$$

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = Q(E\,\hat{\mathbf{z}} + B\dot{z}\,\hat{\mathbf{y}} - B\dot{y}\,\hat{\mathbf{z}}) = m\mathbf{a} = m(\ddot{y}\,\hat{\mathbf{y}} + \ddot{z}\,\hat{\mathbf{z}})$$

(General solution)

$$\hat{Q}\bar{B}\dot{z} = m\ddot{y}$$

$$QE - QB\dot{y} = m\ddot{z}$$

$$\xrightarrow[\text{(Cyclotron frequency)}]{\omega \equiv \frac{QB}{m}}$$

$$\ddot{y} = \omega\dot{z}$$

$$\ddot{z} = \omega\left(\frac{E}{B} - \dot{y}\right)$$

$$\longrightarrow \left.\begin{array}{rcl} y(t) & = & C_1\cos\omega t + C_2\sin\omega t + (E/B)t + C_3, \\ z(t) & = & C_2\cos\omega t - C_1\sin\omega t + C_4. \end{array}\right\}$$

At t = 0: $y(0) = z(0) = 0$

$\dot{y}(0) = \dot{z}(0) = 0$

$$\longrightarrow \quad y(t) = \frac{E}{\omega B}(\omega t - \sin\omega t)$$

$$z(t) = \frac{E}{\omega B}(1 - \cos\omega t)$$

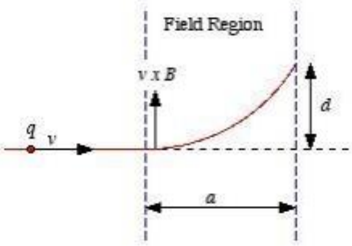
At z=0: $|F_e| = |F_{mag}| \rightarrow$ $\boxed{v = \frac{E}{B} \rightarrow R = \frac{v}{\omega} = \frac{E}{\omega B}}$ $\boxed{\rightarrow}$ $\boxed{\sin^2\omega t + \cos^2\omega t = 1}$ $\boxed{\rightarrow}$ $\boxed{(y - R\omega t)^2 + (z - R)^2 = R^2}$

This is the formula for a *circle,* of radius *R,* whose center (0, *Rωt, R)*travels in the y-direction at a constant speed, v = E/B.

→Cycloid motion

Problems:

1. A particle of charge q enters a region of uniform magnetic field $\vec{B}$ (pointing into the page).The field deflects the particle a distance d above the original line of flight, as shown in figure. Is the charge positive or negative? In terms of a, d, B and q, find the momentum of the particle.



2. Find and sketch the trajectory of the particle, if it starts at the origin with velocity

(a) $\overrightarrow{v(0)}=\frac{E}{B}\hat{y}$

(b)) $\overrightarrow{v(0)}=\frac{E}{2B}\hat{y}$

(c)) $\overrightarrow{v(0)}=\frac{E}{B}(\hat{y}+\hat{z})$