PHY-H-CC-T-03: ELECTRICY AND MAGNETISM

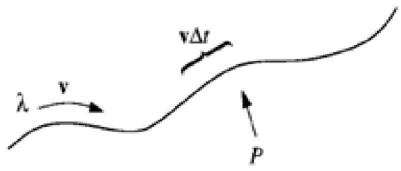
LECTURE-2 (Pabitra Halder (PH), Department of Physics, Berhampore Girls' College)

Last lecture, I discuss about Lorentz force and its application. Now this lecture, I discuss about different current configuration, continuity equation of current density and various technique for finding the magnetic induction or magnetic field vector due to different current configuration.

In analogy with electrostatics we discuss about different current configuration. There are three types of current configuration namely

1. Line current:

When line charge λ travelling down at a velocity v, then current I=λv



The magnetic force on segment of a current carrying wire is

$\vec{F}_{mag}=\int(\vec{v}\times\vec{B})dq=\int(\vec{v}\times\vec{B})\lambda dl=\int(\vec{I}\times\vec{B})dl$

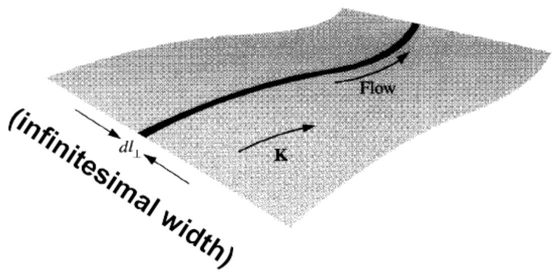
For steady current (constant in magnitude) along the wire

$\vec{F}_{mag}=\mathrm{I}\int(\vec{dl}\times\vec{B})$

I and dl both point in same direction.

2. Surface current:

When charge flows over a surface with surface current density K:



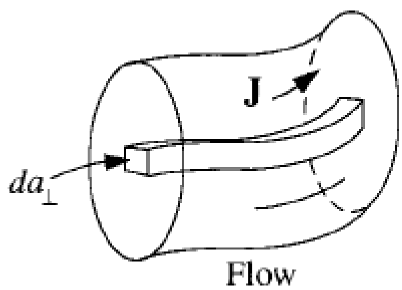
$\vec{K}=\frac{d\vec{I}}{dl_\perp}=\sigma\vec{v}$ → Current per unit width perpendicular to flow

$\vec{F}_{mag}=\int(\vec{v}\times\vec{B})dq=\iint(\vec{v}\times\vec{B})\sigma da$

$\vec{F}_{mag}=\iint(\vec{K}\times\vec{B})da$

3. Volume current:

When charge flow over a volume with a volume current density $\vec{J}$:



$\vec{J}=\frac{d\vec{I}}{da_\perp}=\rho\vec{v}$ → Current per unit area perpendicular to flow

$\vec{F}_{mag}=\int(\vec{v}\times\vec{B})dq=\iiint(\vec{v}\times\vec{B})\rho d\tau$

$\vec{F}_{mag}=\iiint(\vec{J}\times\vec{B})d\tau$

Question:

1. What is abA and ampere? Write down the relation between them.

Ans. abA is defined as the one ab coulomb of charge carrier moving past a specific point in one second.

The ab ampere (abA) is unit of current in the c.g.s system of electromagnetic. The abA is moderately large unit of current, equivalent to 10 ampere.

Alternate definition of abA:

An abA current is the amount of current flowing in a circular path of one centimetre radius produce a magnetic field induction of 2π oersteds at the centre of the circular loop.

Ampere:

Ampere is defined as the one coulomb of charge carrier moving past a specific point in one second.

Relation:

1 abA = 10 amp

ab coulomb:

The amount charge that passes in one second through any cross-section of a conductor carrying steady current of one ab ampere is called one coulomb charge.

It is approximately equal to the charge contained in 6.24×10^{19} electrons (1 electronic charge is about 1.6×10^{19} coulomb).

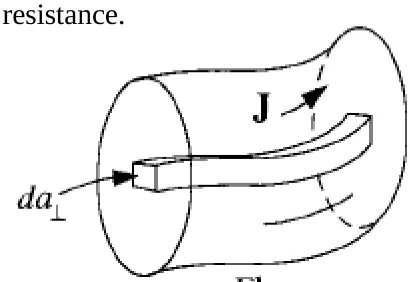
ab henry:

The self-inductance of a circuit or mutual inductance of two circuits in which the variation of current at the rate of one ab ampere per second result in an induced electromotive force of one ab volt is called an one ab henry self-inductance or mutual inductance.

ab ohm:

The resistance of a conductor that with a constant current of one ab ampere through it, maintain between its terminals a potential difference of one ab volt is called an ab ohm resistance.

Continuity equation for current density:



We assume that the charge is flowing through a three-dimensional region.

If the current in infinitesimal cross-section $da_\perp$, running parallel to the flow $d\vec{I}$, then volume current density $\vec{J}=\frac{d\vec{I}}{da_\perp}$

The current crossing a surface S can be written as $I = \int_S J\, da_\perp = \int_S \mathbf{J} \cdot d\mathbf{a}$

The total charge per unit time leaving a volume V is

$$\oint_S \mathbf{J} \cdot d\mathbf{a} = \int_\mathcal{V} (\nabla \cdot \mathbf{J})\, d\tau$$

Because charge is conserved $$\int_{\mathcal{V}} (\nabla \cdot \mathbf{J})\, d\tau = -\frac{d}{dt}\int_{\mathcal{V}} \rho\, d\tau = -\int_{\mathcal{V}} \left(\frac{\partial \rho}{\partial t}\right) d\tau$$

Therefore, $$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$ → Continuity equation (Local charge conservation)

For steady current, $\frac{\partial \rho}{\partial t}$=0, then $\vec{\nabla}.\vec{J}$ =0 →$\oint\!\!\oint \vec{J}.\overrightarrow{ds}$=0 → $\sum_{j=0}^{n} I_j$ =0 → Kirchhoff's current law

Idea of displacement current and electric current density:

From Gauss's theorem in electrostatics, we have

$\iint \vec{E}.\overrightarrow{ds}=\frac{Q_{enclosed}}{\epsilon_0}$ → $\iint \epsilon_0\, \vec{E}.\overrightarrow{ds}= Q_{enclosed}=\iiint \rho\, d\tau$

We define $\vec{D}= \epsilon_0\, \vec{E}$, where $\vec{D}$ is the electric displacement vector.

$\iint \vec{D}.\overrightarrow{ds}= \iiint \rho\, d\tau$

Applying Gauss's divergence theorem, we get

$\iiint(\vec{\nabla}.\vec{D})$ dτ = $\iiint \rho\, d\tau$ →$\frac{\partial}{\partial t}\iiint(\vec{\nabla}.\vec{D})$ dτ = $\frac{\partial}{\partial t}\iiint \rho\, d\tau$→$\iiint(\vec{\nabla}.\frac{\partial \vec{D}}{\partial t})$ dτ - $\iiint \frac{\partial \rho}{\partial t} d\tau$ =0

Since, dτ is an arbitrary,

$\vec{\nabla}.\frac{\partial \vec{D}}{\partial t} = \frac{\partial \rho}{\partial t}$ → $\vec{\nabla}.\vec{J} + \vec{\nabla}.\frac{\partial \vec{D}}{\partial t}$ =0 →$\vec{\nabla}.(\vec{J} + \frac{\partial \vec{D}}{\partial t})$ =0 ……………(1) $since, \left[\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0\right]$

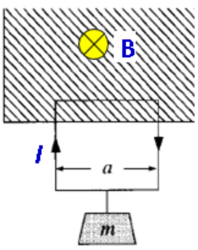
$\vec{J}$ is called the conduction current density while $\frac{\partial \vec{D}}{\partial t}$ is called displacement current density in the circuit.

$\vec{J} + \frac{\partial \vec{D}}{\partial t} = \overrightarrow{J_{eff}}$ = effective current density in the medium.

Then equation (1) implies that the effective current density in the medium is a solenoidal vector.

Problem:

1. For what current *I*, in the loop, would the magnetic force upward exactly balance the gravitational force.



The magnetic forces on the horizontal segment $$\mathbf{F}_{mag} = I\int (d\mathbf{l} \times \mathbf{B}) = IBa$$

Magnetic force on the two vertical segments cancels, then for F_{mag} to balance the weight *(mg)* I= $\frac{mg}{Ba}$.

2. A particle of charge q and mass m is shot in a region of uniform magnetic field $\vec{B}$ pointing in the z-direction with velocity component v_x and v_y in x and y direction respectively. Set up the differential equation of motion in variable v_x and v_y. Show that v_x and v_y vary simple harmonically with time & $v_x^2 + v_y^2$ =const.

Ans. From question we can write, $\vec{B}$ = B $\hat{k}$ and $\vec{v} = \hat{\imath}\, v_x + \hat{\jmath}\, v_y$.

The magnetic force acting on the charge particle $\vec{F_m}$= q ($\vec{v} \times \vec{B}$), then from Newton's 2nd law we can write –

m$\frac{d\vec{v}}{dt}$ = q ($\hat{\imath}\, v_y B$ - $\hat{\jmath}\, v_x B$) → m $\frac{d}{dt}$($\hat{\imath}\, v_x + \hat{\jmath}\, v_y$) = q ($\hat{\imath}\, v_y B$ - $\hat{\jmath}\, v_x B$)

Therefore, m$\frac{dv_x}{dt}$ = q $v_y B$ & m$\frac{dv_y}{dt}$ =- q $v_x B$

$\frac{dv_x}{dt}$ = ωv_y ·········(i) & $\frac{dv_y}{dt}$ = - ωv_x ··········(ii), where ω= qB

(i) + (ii)×j we get,

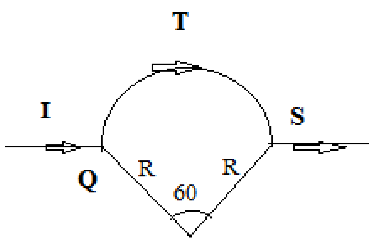
$\frac{d}{dt}$(v_x + j v_y) = - jω (v_x+j v_y) → $\frac{dZ}{dt}$ = - j ω Z → Z= C$e^{-j\omega t}$, where j =$\sqrt{-1}$ and Z= v_x+j v_y

→ v_x+j v_y = C ($\cos\omega t$ - j $\sin\omega t$), where C an arbitrary constant.

Therefore, v_x = C $\cos\omega t$ & v_y = - C $\sin\omega t$ → v_x and v_y vary simple harmonically

$v_x^2 + v_y^2 = C^2$ = const.

3. A circular arc QTS is kept in an external magnetic field $\vec{B_0}$ as shown in figure. The arc carries a current I. The magnetic field is directed normal and into the page. Find the force acting on the arc?



Ans. The amount of force acting on the arc $\vec{F}$ = I ($\vec{l} \times \vec{B}$) = I$lB_0 \sin\theta$

$$d\vec{F} = I\, B_0 \sin\theta\, dl\, \hat{k}, \text{ Now, } dl=R\, d\theta$$

$$d\vec{F} = I\, B_0 \sin\theta\, R\, d\theta\, \hat{k}$$

$$\vec{F} = \int_{60}^{120} I\, B_0 \sin\theta\, R\, d\theta\, \hat{k} = IB_0\, R\, \hat{k}$$

To be continued···············