PHY-H-CC-T-03: ELECTRICY AND MAGNETISM

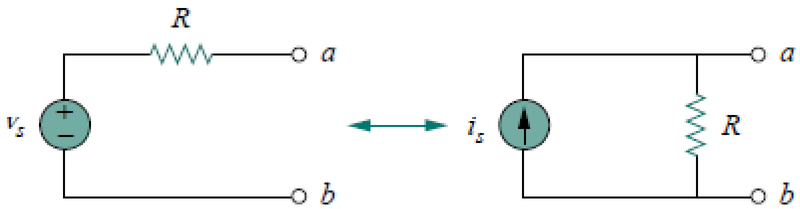
LECTURE-6 (Pabitra Halder, Assistant Professor, Department of Physics, Berhampore Girls' College)

<u>NETWORK THEORY</u>

<u>Source transformation:</u>

Source transformation is the tool for simplifying circuits. Basic to these tools is the concept of equivalence. An equivalent circuit is one whose v-i characteristics are identical with the original circuit.

A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source is in parallel with a resistor R, or vice versa.

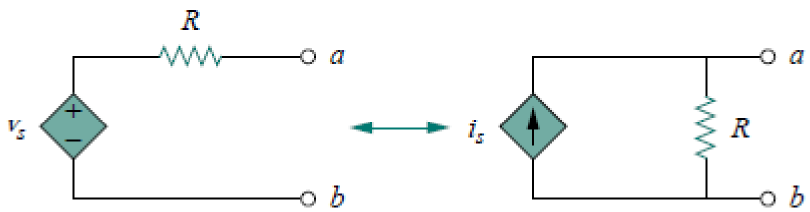


Transformation of independent sources.

The two circuits in above figure are equivalent. Since, they have the same voltage-current relation at terminals a-b. If the sources are turned off, the equivalent resistance at terminals a-b in both circuits is R. Also, when terminals a-b are short circuited, the short-circuit current flowing from a to b is $i_{sc}=\frac{v_s}{R}$ in the circuit on the left-hand side and $i_{sc}=i_s$ for the circuit on the righthand side. Thus, $\frac{v_s}{R}=i_s$ in order for the two circuits to be equivalent. Hence, source transformation requires that

$$v_s = i_s R \text{ or } i_s = \frac{v_s}{R}$$

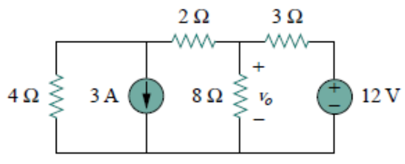
Source transformation also applies to dependent sources, provided we carefully handle the dependent variable. As shown in figure below, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa.



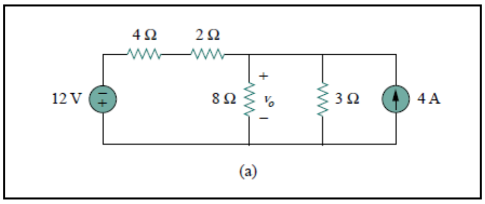
Transformation of dependent sources.

<u>Example:</u>

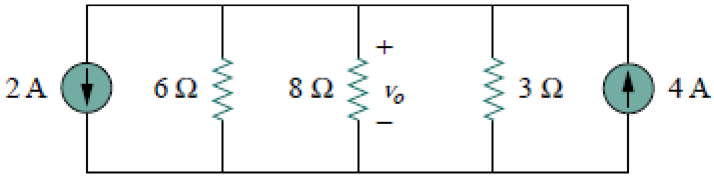
Use source transformation to find $v_0$in the circuit in figure beside.



Ans. We first transform the current and voltage sources to obtain the circuit in figure (a)

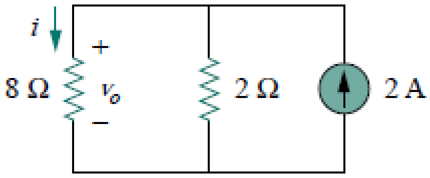


Combining the 4Ω and 2Ω resistors in series and transforming the 12-V voltage source gives us figure (b)



(b)

We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in figure (c).



(c)

We use current division in figure (c) to get,

$$i = \frac{2}{2+8} \times 2 = 0.4 \text{ A and } v_0 = 8\, i = 8\times 0.4 = 3.2 \text{ V}$$

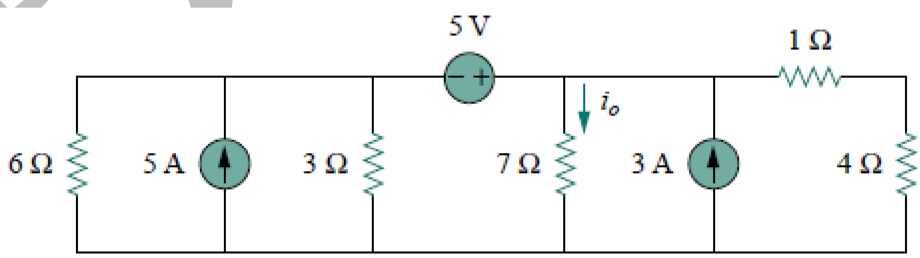
Alternatively, since the 8Ω and 2Ω resistors in figure (c) are in parallel, they have the same voltage v_0 across them. Hence,

$$v_0 = (8\|2)(2 \text{ A}) = \frac{8\times 2}{8+2}(2) = 3.2 \text{ V}$$

<u>Problem:</u>

Find i_0 in the circuit of figure below using source transformation.

Ans. 1.78 A

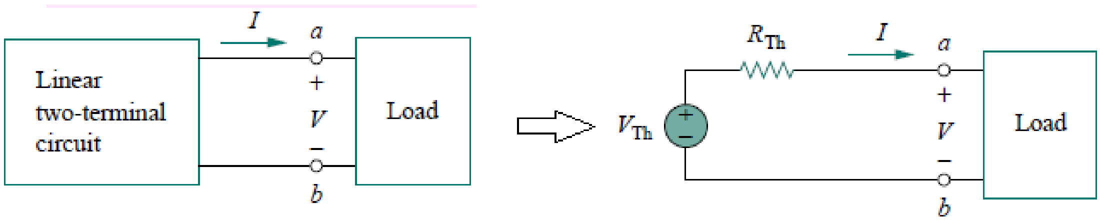


<u>Thevenin's Theorem:</u>

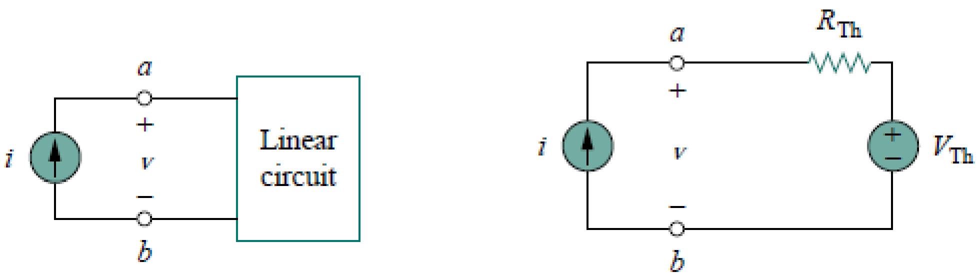
Thevenin equivalent circuit was developed in1883 by M.LeonThevenin(1857–1926), a French telegraph engineer.

Statement:

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where $V_{Th}$is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



<u>Derivation of Thevenin's Theorem:</u>



(a) (b)

Consider the linear circuit in figure (a). It is assumed that the circuit contains resistors, and dependent and independent sources. We have access to the circuit via terminals a and b, through which current from an external source is applied. Our objective is to ensure that the voltage-current relation at terminals a and b is identical to that of the Thevenin equivalent in figure (b). For the sake of simplicity, suppose the linear circuit in figure (a) contains two independent voltage sources v_{s1} and v_{s2} and two independent current sources $i_{s1}$and i_{s2}. We may obtain any circuit variable, such as the terminal voltage *v*, by applying superposition. That is, we consider the contribution due to each independent source including the external source *i*. By superposition, the terminal voltage *v* is

$$v = A_0\, i + A_1\, v_{s1} + A_2\, v_{s2} + A_3\, i_{s1} + A_4\, i_{s2} \quad \cdots\cdots\cdots\cdots (1)$$

where A_0,A_1,A_2,A_3, and A_4 are constants. Each term on the right-hand side of equation (1) is the contribution of the related independent source; that is, $A_0\, i$ is the contribution to v due to the external current source *i*, $A_1\, v_{s1}$ is the contribution due to the voltage source v_{s1}, and so on. We may collect terms for the internal independent sources together as B0, so that equation (1) becomes

$$v = A_0\, i + B_0 \quad \cdots\cdots\cdots\cdots (2)\,,$$

where $B_0 = A_1\, v_{s1} + A_2\, v_{s2} + A_3\, i_{s1} + A_4\, i_{s2}$. We now want to evaluate the values of constants A_0 and B_0. When the terminals a and b are opencircuited, $i = 0$ and $v = B_0$. Thus B_0 is the open-circuit voltage v_{oc}, which is the same as V_{Th}, so

$$B_0 = V_{Th} \quad \cdots\cdots\cdots\cdots (3)$$

When all the internal sources are turned off, $B_0 = 0$. The circuit can then be replaced by an equivalent resistance Req, which is the same as R_{Th}, and equation (2) becomes

$$v = A_0\, i = R_{Th}\, i \rightarrow A_0 = R_{Th} \quad \cdots\cdots\cdots\cdots (4)$$

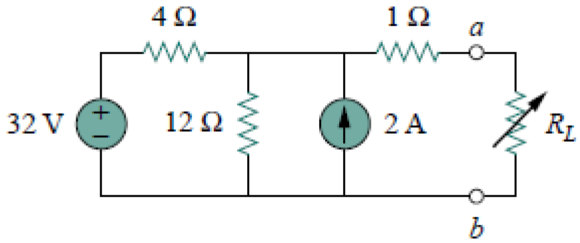
Substituting the values of A_0 and B_0 in equation (2) gives

$$v = R_{Th}\, i + V_{Th} \quad \cdots\cdots\cdots\cdots (5)$$

which expresses the voltage-current relation at terminals a and b of the circuit in figure (b). Thus, the two circuits in figure (a) and (b) are equivalent.

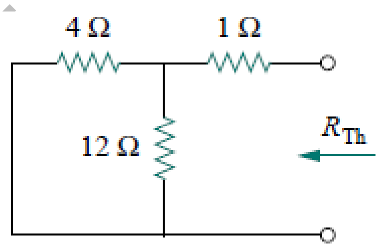
Examples:

1. Find the Thevenin equivalent circuit of the circuit shown in figure below, to the left of the terminals a-b. Then find the current through RL = 6Ω,16Ω, and 36Ω.



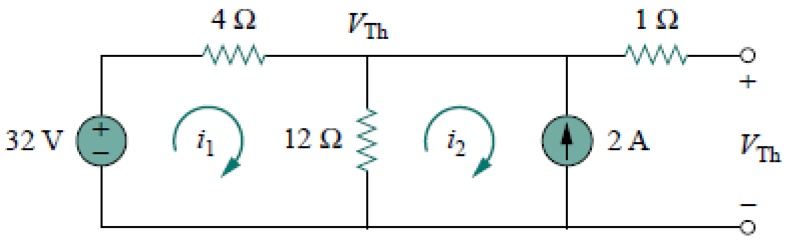
Ans. We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in figure (a). Thus,

$R_{Th} = (4 \parallel 12) + 1 = \frac{4\times12}{4+12} + 1 = 4\Omega$



(a)

To find V_{Th}, consider the circuit in figure (b). Applying mesh analysis to the two loops, we obtain



(b)

For loop (1), -32 + 4i_1 + 12(i_1-i_2) =0 and for loop (2), i_2= -2 A

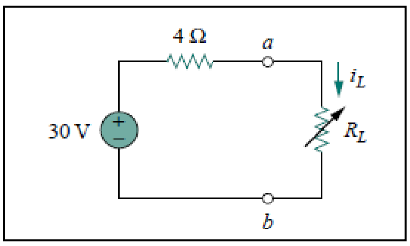
Solving for i_1, we get i_1= 0.5 A. Thus,

V_{Th} = 12 (i_1-i_2) = 12×(0.5 +2) V = 30 V

The Thevenin equivalent circuit is shown in figure below,

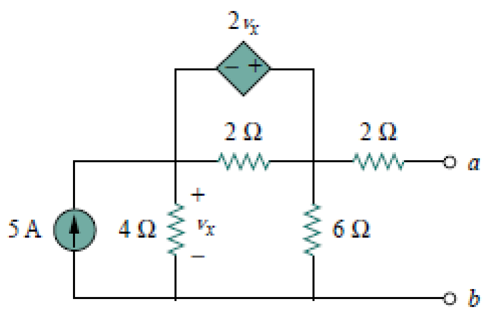
The current through R_L is

$i_L = \frac{V_{Th}}{R_{Th}+R_L} = \frac{30}{4+R_L}$

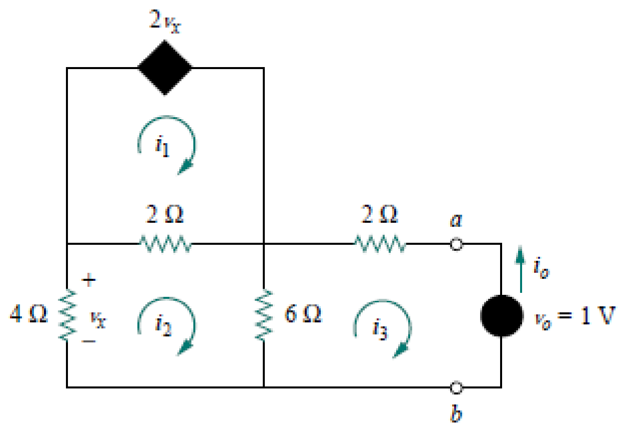


When R_L = 6Ω,i_L = $\frac{30}{4+6}$ = 3 A. When R_L = 6Ω, i_L = $\frac{30}{4+16}$ = 1.5A and when R_L= 36Ω, i_L=$\frac{30}{4+36}$ = 0.75A

2. Find the Thevenin equivalent of the circuit in figure.



Ans. This circuit contains a dependent source, unlike the circuit in the previous example. To find R_{Th}, we set the independent source equal to zero but leave the dependent source alone. Because of the presence of the dependent source, however, we excite the network with a voltage source vo connected to the terminals as indicated in figure (a). We may set v_0= 1 V to ease calculation, since the circuit is linear. Our goal is to find the current io through the terminals, and then obtain R_{Th}=$\frac{1}{i_0}$. (Alternatively, we may insert a 1-A current source, find the corresponding voltage vo, and obtain R_{Th}=$\frac{v_0}{1}$.)



(a)

ApplyingmeshanalysistoIoop1inthecircuitinFig.4.32(a)results in

-2v_x + 2(i_1- i_2) =0 → v_x = i_1- i_2

But, -4i_2 = v_x = i_1- i_2 → i_1 = -3 i_2 ············ (1)

For loops 2 and 3, applying KVL produces

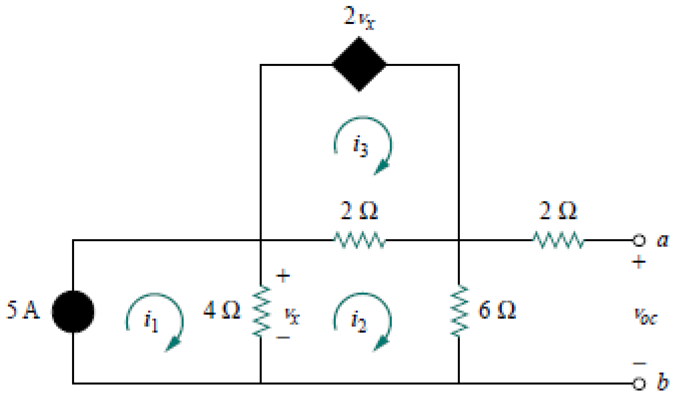
4 i_2 + 2 (i_2 - i_1) + 6 (i_2 - i_3) = 0 ················· (2) and 2 i_3 + 1+ 6 (i_3 - i_2) = 0 ················· (3)

Solving these equations gives

i_3 = - $\frac{1}{6}$ A. But i_0 =− i_3 = 1/6 A. Hence,

R_{Th}=$\frac{1\,V}{i_0}$ = 6 Ω

To get V_{Th}, we find v_{oc} in the circuit of figure (b). Applyingmesh analysis, we get



(b)

For mesh 1

i_1 = 5 A ············· (4)

For mesh 2

6 i_2 + 2 (i_2 - i_3) + 4 (i_2 -i_1) = 0 → 12 i_2 - 4 i_1 - 2 i_3 = 0 ············· (5)

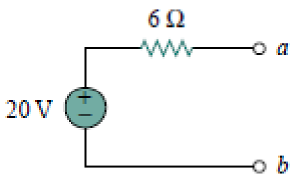
For mesh 3

-2 v_x + 2 (i_3 - i_2) = 0 ············· (6)

But 4 (i_1 -i_2) = v_x. Solving these equations leads to i_2 = 10/3 A. Hence,

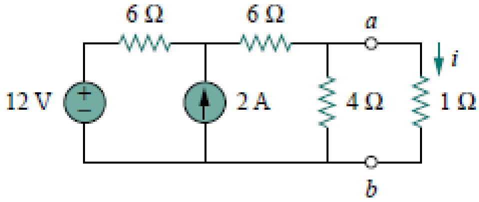
V_{Th} = v_{oc} = 6 i_2 = 20 V

The Thevenin equivalent is as shown in figure below.



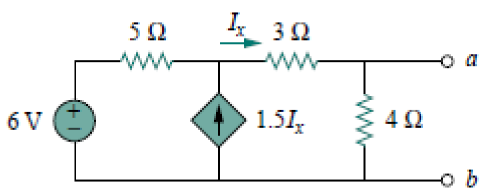
<u>Problems:</u>

1. Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit in figure below. Then find i.



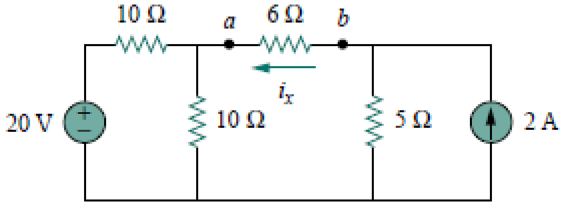
Ans. V_{Th} = 6 V, R_{Th} = 3 Ω, i = 1.5 A.

2. Find the Thevenin equivalent circuit of the circuit in figure below to the left of the terminals.



Ans. V_{Th} = 5.33 V, R_{Th} = 0.44Ω

3. Find the Thevenin equivalent looking into terminals a-b of the circuit in figure and solve for i_x.



4. For the circuit in figure below, obtain the Thevenin equivalent as seen from terminals:

(a) a-b (b) b-c