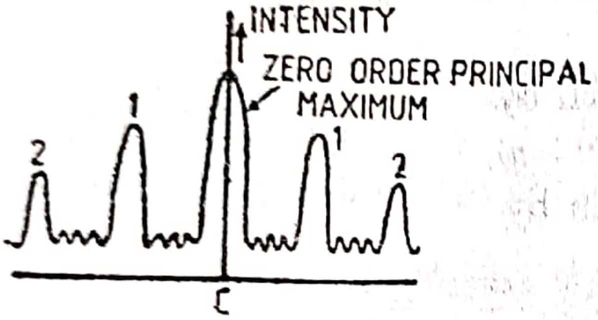
of the central maximum.

7.7. Fraunhofer diffraction at N slits

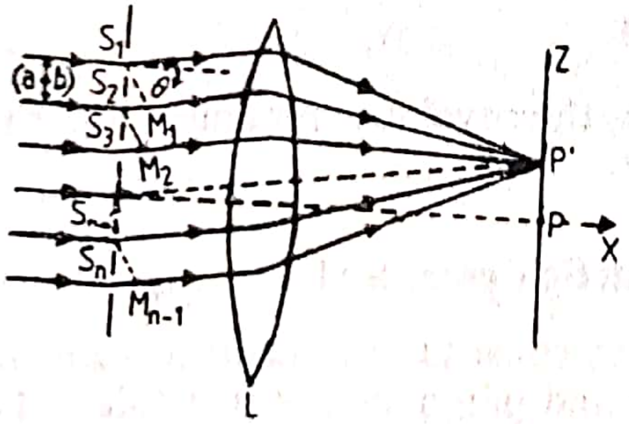
$S_1, S_2, .., S_n \rightarrow N$ parallel slits, each of width 'a' and separated by opaque space 'b'.
$L \rightarrow$ a collimating lens.
$Z \rightarrow$ screen.

Result : The pattern obtained on the screen is called the Fraunhofer diffraction pattern due to N parallel equidistant slits. The pattern consists of a central maximum called **Zero order principal maximum** followed by a number of principal maxima on either side. There are $(N-1)$ equi-spaced minima between any two consecutive principal maxima.



There are $(N-2)$ other maxima coming alternatively with the minima between two consecutive principal maxima. These maxima are called **secondary maxima.** The intensity distribution is shown in the figure.

Theory : A plane wavefront of monochromatic light is incident normally on N parallel slits. The light is diffracted through N slits and is focussed by a lens L on the screen Z placed in the focal plane of L The wave diffracted from all the slits in direction θ are equivalent to N parallel waves, each wave starting from mid-points, S_1, S_2,....S_n of the slits.



Let S_1M_1, S_2M_2...... $S_{n-1}M_{n-1}$ be perpendiculars dropped from S_1, S_2,... S_{n-1} on to the diffracted rays. The path difference between the waves from

S_1 and S_2 is $\quad S_2M_1 = (a+b)\sin\theta = d\sin\theta,$
S_2 and S_3 is $\quad S_3M_2 = d\sin\theta,$
S_{n-1} and S_n is $\quad S_nM_{n-1} = d\sin\theta,$

Thus as we pass from one wave to another, the path goes on increasing by the same amount $d\sin\theta$.

To obtain an expression for the intensity distribution due to diffraction at N slits, the expression for instantaneous displacement (dZ) has to be integrated for N slits.

For a single slit,

$$Z=k\int_{-a/2}^{a/2} \sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{y\sin\theta}{\lambda}\right) dy=k\int_{-a/2}^{a/2} \varphi(y)dy$$

when $\quad \varphi(y)=\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{y\sin\theta}{\lambda}\right).$

For N slits :

$$Z=k\int_{-a/2}^{a/2} \varphi(y)\, dy+\int_{d-a/2}^{d+a/2} \varphi(y)dy$$

$$+\int_{2d-a/2}^{2d+a/2} (y)\, dy+\ldots.+\int_{(N-1)d-a/2}^{(N-1)d+a/2} \varphi(y)\, dy$$

$$=ka\frac{\sin\alpha}{\alpha}\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)\right.$$

$$+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{2d\sin\theta}{\lambda}\right)+\ldots.$$

$$\left.+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{\overline{N-1}\, d\sin\theta}{\lambda}\right)\right]$$

$$=ka\frac{\sin\alpha}{\alpha}\cdot\frac{\sin N\beta}{\sin\beta}\cdot\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{\overline{N-1}\, d\sin\theta}{2\lambda}\right)$$

where $\alpha=\pi a\sin\theta/\lambda$, and $\beta=\pi d\sin\theta/\lambda$.

The intensity at a point P' is given by

$$I=k^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)=I_0\,\frac{\sin^2\alpha}{\alpha^2}\cdot\frac{\sin^2 N\beta}{\sin^2\beta}\quad\left[\because\quad I_0=k^2a^2\right].$$

The factor ($\sin^2\alpha/\alpha^2$) represents the diffraction pattern due to a single slit. The factor ($\sin^2 N\beta/\sin^2\beta$) represents the interference effects due to the secondary waves from the N slits.

7.8. (i) Principal Maxima

$$\underset{\beta\to 0}{\text{Lt}} \frac{\sin N\beta}{\sin\beta} = N.$$

When $\beta\to 0$, the resultant intensity of maxima is

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2}.\ N^2 = I_0 N^2 . \frac{\sin^2\alpha}{\alpha^2}.$$

Thus if we increase the number of slits, the intensity of principal maxima increases. The directions of principal maxima are given by

$$\sin\beta = 0$$

$$\Rightarrow \quad \beta = \pm n\pi \text{ where } n = 0, 1, 2, \ldots.$$

$$\Rightarrow \frac{\pi d \sin\theta}{\lambda} = \pm n\pi$$

$$\Rightarrow \quad d\sin\theta = (a+b) \sin\theta = \pm n\lambda.$$

When $n=0$, we get $\theta=0$. This is the direction in which all the waves arrive in the same phase and hence we get a bright central maximum. This is called the **Zero order principal maximum.** By putting $n=1, 2, 3, \ldots$ we obtain first, second, third....order principal maxima respectively.

(ii) **Minima** : For minima, we must have

$$\sin N\beta = 0 \qquad (\because \quad \sin\beta \neq 0)$$

$$\Rightarrow \qquad N\beta = \pm m\pi \quad \Rightarrow N\frac{\pi d \sin\theta}{\lambda} = \pm m\pi$$

$$\Rightarrow \quad N(a+b) \sin\theta = \pm m\lambda.$$

Here n can have all integral values except $0, N, 2N, 3N, \ldots$ which give $\sin\beta = 0$ which is the condition for principal maxima. The positive and negative values of m indicate that the minima of a given order lie symmetrically on either side of the central principal maximum.

N.B. $m=0$ gives principal maximum of zero order. $m=1, 2, 3, \ldots (N-1)$ give $(N-1)$ minima, $m=N$ gives principal maximum of first order.

(iii) Secondary Maxima

Between two consecutive principal maxima there are $(N-1)$ minima and hence $(N-2)$ maxima. These maxima are called **secondary maxima.** Their positions are obtained by

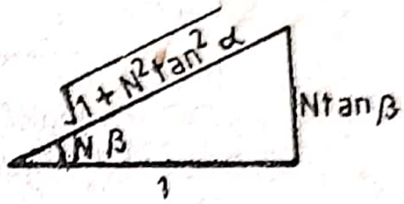
$$\Rightarrow I_0 \frac{\sin^2\alpha}{\alpha^2}.2\left(\frac{\sin N\beta}{\sin\beta}\right).\frac{\sin\beta.\ N\cos N\beta-\sin N\beta.\ \cos\beta}{\sin^2\beta}=0$$

$$\Rightarrow N\cos N\beta.\ \sin\beta-\sin N\beta.\ \cos\beta=0$$

$$\Rightarrow \tan N\beta=N\tan\beta.$$

The roots of this equation (other than $\beta=\pm n\pi$) give the position of secondary maxima.

In order to find the intensity of secondary maxima, we have



$$\sin N\beta=N\tan\beta/\sqrt{1+N^2\tan^2\beta}$$

$$\Rightarrow \sin^2 N\beta=N^2\tan^2\beta/\ (1+N^2\tan^2\beta)$$

$$\Rightarrow \frac{\sin^2 N\beta}{\sin^2\beta}=\frac{N^2\tan^2\beta}{(1+N^2\tan^2\beta)\ \sin^2\beta}=\frac{N^2}{1+(N^2-1)\sin^2\beta}.$$

Hence the intensity of secondary maxima is given by

$$I_s=I_0\frac{\sin^2\alpha}{\alpha^2}.\frac{N^2}{1+(N^2-1)\sin^2\beta}.$$

Thus as N increases the intensity of secondary maxima decreases. When N is very large (Plane grating), the secondary maxima are not visible. There is uniform darkness between any two consecutive principal maxima.