CHAPTER 7

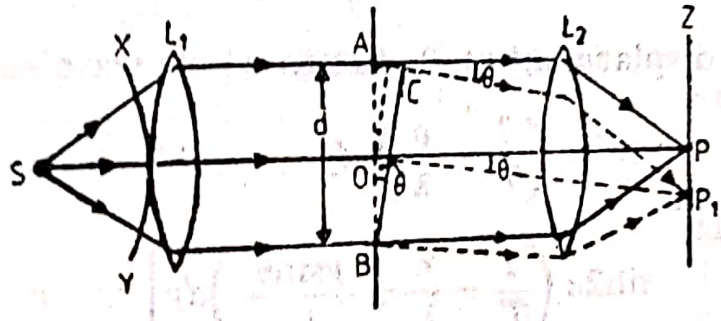
FRAUNHOFER DIFFRACTION

7.1. Fraunhofer Diffraction

It arises when the source of light and the screen are effectively at infinite distance from the diffracting aperture. The incident wave-front is plane and the diffracted wavefront is also plane.

7.2. Fraunhofer diffraction at a single slit

$S \rightarrow$ a narrow slit perpendicular to the plane of paper and illuminated by monochromatic light of wavelength λ.



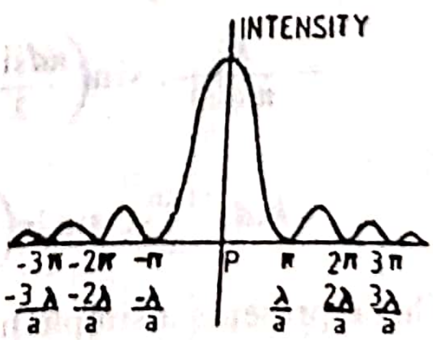
$L_1 \rightarrow$ a collimating lens.

$AB \rightarrow$ a slit of width d.

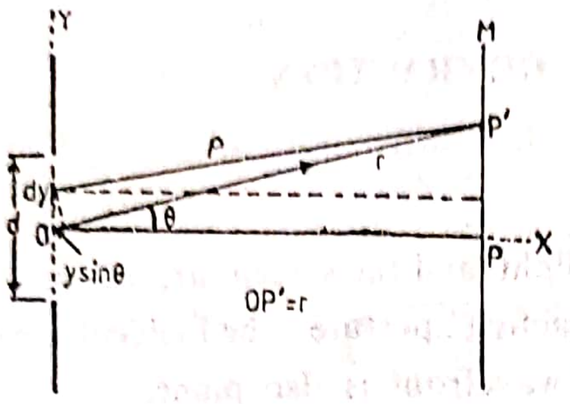
$L_2 \rightarrow$ another lens.

$Z \rightarrow$ a screen perpendicular to plane of paper.

Results : The diffraction pattern consists of a central bright maximum at P followed by secondary maxima and minima on either side. The points on the screen for which the path difference $(AP_1 - BP_1)$ is $n\lambda$ correspond to the positions of secondary minima. The secondary maxima are much less intense and the intensity falls off rapidly from the point P onwards. The intensity distribution on the screen is shown in the figure.



Theory : The centre of the slit O is taken as the origin of the co-ordinate system with x as abscissa and y as ordinate. The slit is divided into a large number of infinitesimal elements of equal width dy. Consider one element dy at a distance y above the origin. Consider a point P' defined by a possible diffracted beam making an angle θ with the normal to the plane of slit. Let $OP'=r$ and the element dy is at a distance ρ from P'.



The displacement at P' due to the element dy at any time t is given by

$$dZ=k.\,dy.\,\sin 2\pi\left(\frac{t}{T}-\frac{\rho}{\lambda}\right) \quad \cdot\cdot \quad \text{(i)}.$$

The resultant displacment at P' due to whole wavefront is :

$$Z=k\int_{-d/2}^{d/2}\sin 2\pi\left(\frac{t}{T}-\frac{\rho}{\lambda}\right)dy$$

$$=k\int_{-d/2}^{d/2}\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{y\sin\theta}{\lambda}\right)dy\left[\because \quad \rho=r-y\sin\theta\right]$$

$$=\frac{-k\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right)\right.$$

$$\left.-\cos 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}-\frac{d\sin\theta}{2\lambda}\right)\right]$$

$$=\frac{-k\lambda}{2\pi\sin\theta}\left[2\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right).\,\sin 2\pi\left(-\frac{d\sin\theta}{2\lambda}\right)\right]$$

$$=\frac{k\lambda}{\pi\sin\theta}.\,\sin\left(\frac{\pi d\sin\theta}{\lambda}\right).\,\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)$$

$$=k.d.\,\frac{\sin\alpha}{\alpha}.\,\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right);\ \left(\because \quad \alpha=\frac{\pi d\sin.\theta}{\lambda}\right).$$

This represents a simple harmonic vibration at P' with amplitude

$$A=k.d.\left(\frac{\sin\alpha}{\alpha}\right).$$

and intensity at P' is given by :

$$I' = k^2d^2\left(\frac{\sin^2\alpha}{\alpha^2}\right) = I_0 . \frac{\sin^2\alpha}{\alpha^2} \qquad (\because \; I_0 = k^2d^2).$$

7.3. (i) Central Maximum

For the point P on the screen, $\theta = 0°$ and hence $\alpha = 0°$. But $\lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$. Hence the Intensity at P is

$$I' = I_0 \text{ (maximum)}.$$

(ii) Minima

The minima occur when—

$$\sin\alpha = 0$$

i.e. when $\qquad \alpha = n\pi \qquad (n = \pm 1, 2, 3, ..)$

or when $\qquad d\sin\theta = n\lambda.$

There are a series of equi-spaced minima on either side of the central maximum.

(iii) Secondary Maxima

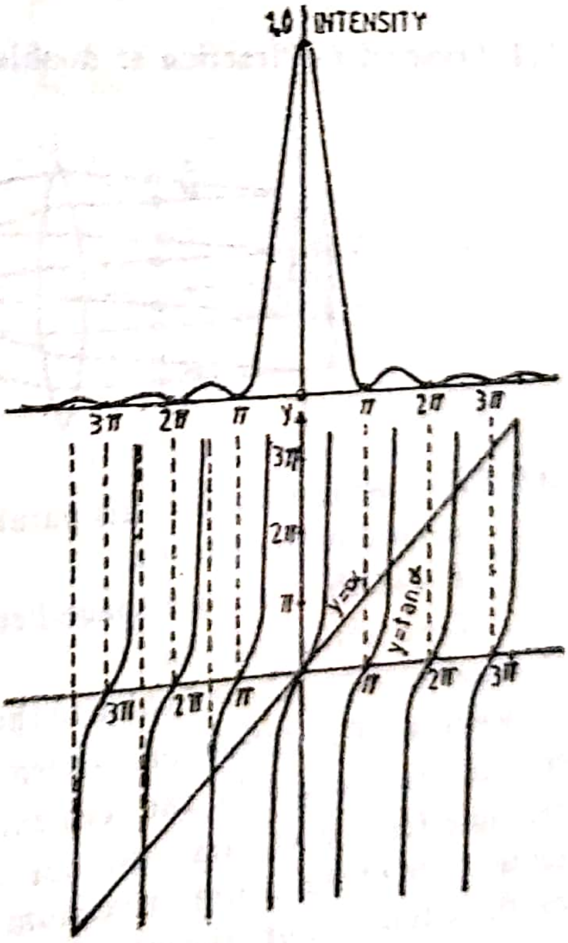
Between the minima there are a series of maxima known as secondary maxima. Their position is found from

$$\frac{dI}{d\alpha} = 0$$

$$\Rightarrow \frac{2\sin\alpha\cos\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^3} = 0$$

$$\Rightarrow \frac{\sin\alpha}{\alpha^2}\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha}\right) = 0$$

$$\Rightarrow \tan\alpha = \alpha.$$



This equation gives the position of maxima. It can be solved graphically by plotting $y = \alpha$ and $y = \tan\alpha$ on the same graph. The first curve is a straight line making an angle of 45° with the axes. The other curves constitute a family with asymptotes at

$$\alpha = \pm\frac{\pi}{2}, \quad \pm\frac{3\pi}{2}, \quad \pm\frac{5\pi}{2}, \ldots.$$

The abscissa of the points of intersection are the roots of equation $\tan\alpha = \alpha$. The exact values of roots are $1{\cdot}430\pi$, $2{\cdot}459\pi$, $3{\cdot}471\pi$, $4{\cdot}477\pi$. They correspond to the positions of **secondary maxima** which are not exactly midway between two minima but are slightly towards the central maximum.

The intensities of the secondary maxima are calculated approximately by evaluating $(\sin^2\alpha/\alpha^2)$ at the half-way positions i.e. at

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots$$

$$I_1 = I_0 \frac{\sin^2(3\pi/2)}{(3\pi/2)^2} = \frac{4}{9\pi^2} \cdot I_0 = \frac{I_0}{22};$$

$$I_2 = I_0 \frac{\sin^2(5\pi/2)}{(5\pi/2)^2} = \frac{4}{25\pi^2} \cdot I_0 = \frac{I_0}{62} \text{ and so on.}$$

Thus the secondary maxima are of decreasing intensity.