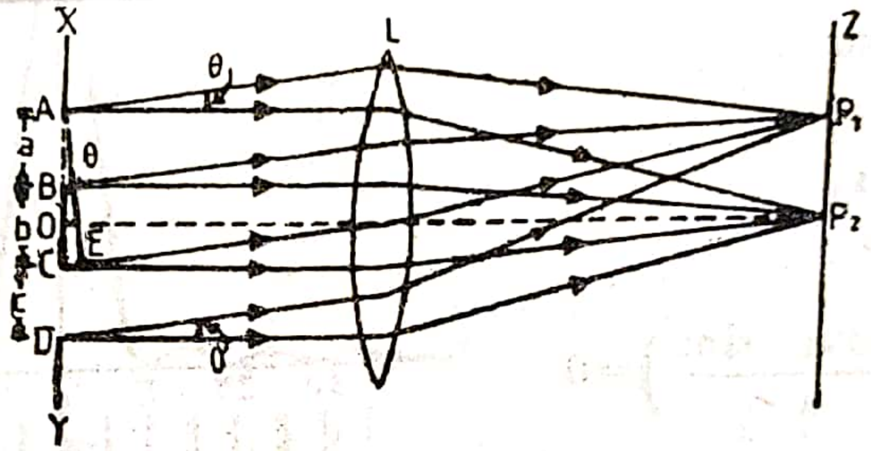
7.4. Fraunhofer diffraction at double slit



AB, *CD*→two rectangular slits parallel to each other.

a→width of each slit.

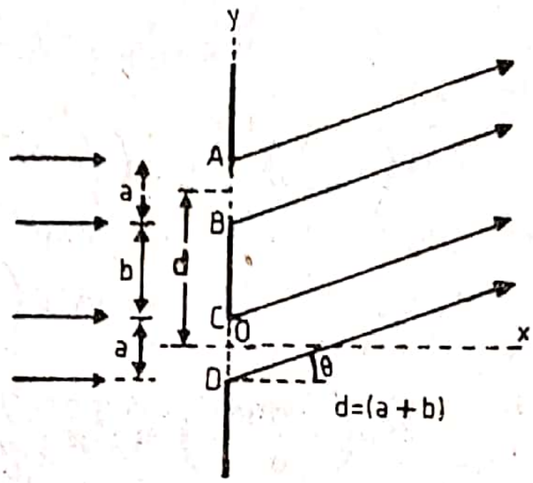
b→width of opaque space between the slits.

L→a collecting lens.

Z→screen perpendicular to the plane of paper.

Results : The diffraction pattern consists of equally spaced interference fringes within the central maximum. The intensities of the interference maxima are not constant but decrease to zero on either side of the central maximum. These maxima reappear two or three times before the intensity becomes too low to be observed.

Theory : The origin is taken at the centre of the slit CD. The element dy is taken at a distance y from O in CD. The limits of integration for CD will be from $-\frac{a}{2}$ to $+\frac{a}{2}$ and those for AB will be from $d-\frac{a}{2}$ to $d+\frac{a}{2}$ where $d=a+b$.



The displacement at P' due to dy at time t is

$$dZ = k\,dy \sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{y\sin\theta}{\lambda}\right).$$

Hence the total displacement due to both the slits is

$$Z = k\left[\int_{-a/2}^{a/2} \sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{y\sin\theta}{\lambda}\right) dy\right.$$

$$\left. + \int_{d-a/2}^{d+a/2} \sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{y\sin\theta}{\lambda}\right) dy\right]$$

$$= ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right)$$

$$-\frac{k\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{y\sin\theta}{\lambda}\right)\right]_{d-a/2}^{d+a/2}$$

$$= ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right)$$

$$-\frac{k\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{d\sin\theta}{\lambda} + \frac{a\sin\theta}{2\lambda}\right)\right.$$

$$\left. - \cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{d\sin\theta}{\lambda} - \frac{a\sin\theta}{2\lambda}\right)\right]$$

$$= ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right)$$

$$+\frac{k\lambda}{\pi\sin\theta}\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right).\ \sin\left(\frac{\pi a\sin\theta}{\lambda}\right)\right]$$

$$=ka\left(\frac{\sin\alpha}{\alpha}\right)\left[\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}\right)\right.$$

$$\left.+\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{\lambda}\right)\right]\left(\because\quad \alpha=\frac{\pi a\sin\theta}{\lambda}\right)$$

$$=2ka\left(\frac{\sin\alpha}{\alpha}\right)\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right).\ \cos\left(\frac{\pi d\sin\theta}{\lambda}\right)$$

$$=2ka\left(\frac{\sin\alpha}{\alpha}\right)\cos\beta.\sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right)\left(\because\quad \beta=\frac{\pi d\sin\theta}{\lambda}\right)$$

$$=A.\ \sin 2\pi\left(\frac{t}{T}-\frac{r}{\lambda}+\frac{d\sin\theta}{2\lambda}\right).$$

Hence the resultant amplitude is

$$A=2ka\left(\frac{\sin\alpha}{\alpha}\right)\cos\beta.$$

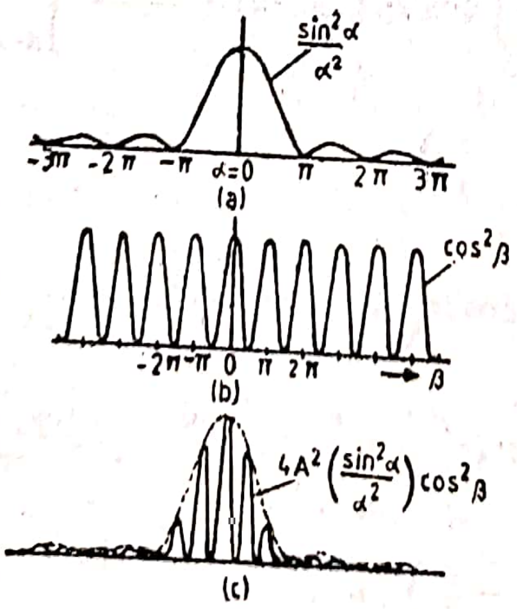
Hence the intensity at P' is given by

$$I=A^2=4k^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right).\cos^2\beta=4I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right).\ \cos^2\beta.$$

For central maximum, $\theta=0°$ and hence $\alpha=0°$ and $\beta=0°$. Hence, the intensity of central maximum $=4I_0$.

The intensity in diffraction pattern is given by the product of two variable factors. The first factor $(\sin^2\alpha/\alpha^2)$ gives the diffraction pattern of a single slit. The second factor $\cos^2\beta$ is characteristic of the interference pattern produced by two slits. The resultant intensity is zero when either of the factors is zero. This occurs when $\alpha=\pi, 2\pi, 3\pi, \ldots$

or when $\beta=\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$



The intensity distribution due to Fraunhofer diffraction at two parallel slits is shown. The full line represents equi-spaced interference fringes.

The dotted curve represents the diffraction maxima and minima. The intensity of the central interference maxima is four times the intensity of the central maximum of the single slit diffraction pattern. The intensity of other interference maxima on the two sides of the central maximum gradually decreases.

7.5. (i) Effect of increasing the slit-width

If we increase the slit-width a, the envelope of the fringe pattern changes so that the central peak is sharper. The fringe-width, which depends on the slit separation (b), does not change.

(ii) Effect of increasing the distance between slits

If the width of slit a is kept constant and the separation b between them is increased, the fringes become closer together, the envelope of the pattern remaining unchanged.

The interference maxima are given by

$$(a+b)\sin\theta = n\lambda$$

and the diffraction minima are given by

$$a\sin\theta = m\lambda; \qquad \therefore \quad \frac{a+b}{a} = \frac{n}{m}.$$

If $a=b$, then $n=2m=2, 4, 6, \ldots$ ($\because$ $m=1, 2, 3, \ldots$).

Thus, the 2nd, 4th, 6th,.....order interference maxima will be absent as they will coincide with 1st, 2nd, 3rd,....order diffraction minima.

If $b=2a$ then $n=3.m=3, 6, 9, \ldots$ ($\because$ $m=1, 2, 3, \ldots$).

Thus, the 3rd, 6th, 9th...order interference maxima will coincide with 1st, 2nd, 3rd,.....order diffraction minima.

(iii) Effect of increasing wavelength

On increasing the wavelength, the envelope becomes broader, and the fringes move further apart.

7.6. Single slit and double slit diffraction compared

The single slit diffraction pattern consists of a central principal maximum with secondary maxima and minima on either side. The secondary maxima and minima are of gradually decreasing intensity.

The double slit diffraction pattern consists of equi-spaced interference maxima and minima within the central maximum. The intensity of central maximum in the diffraction pattern due to a double slit is four times that of the central maximum due to diffraction at single slit. The spacing of the diffraction maxima and the minima depends on the width of the slit (a). The spacing of the interference maxima and minima depends on $(a+b)$ where b is the opaque spacing between the two slits. The intensities of the interference maxima are not constant but decrease to zero on either side of the central maximum.