PHY-H-CC-T-03: ELECTRICY AND MAGNETISM

LECTURE-7 (Pabitra Halder, Assistant Professor, Department of Physics, Berhampore Girls' College)

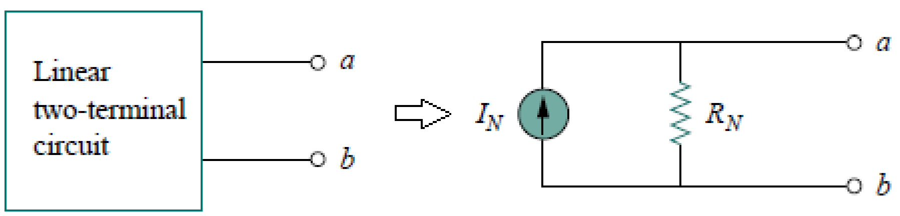
NETWORK THEORY

Norton's Theorem:

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Statement:

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

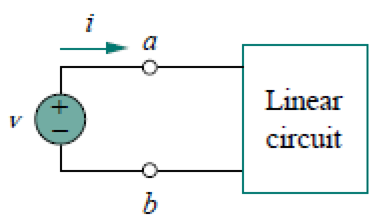


Derivation of Norton's theorem:

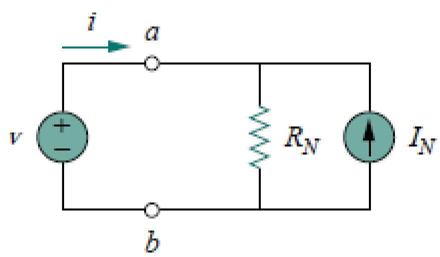
When a linear circuit is driven by a voltage source v as shown in figure (a), the current flowing into the circuit can be obtained by superposition as

$$i = C_0\, v + D_0 \cdots\cdots\cdots\cdots (1)$$

Where $C_0\, v$ is the contribution to i due to the external voltage source v and D_0 contains the contributions to i due to all internal independent sources.



(a)



(b)

When the terminals a-b are short-circuited, $v = 0$ so that $i = D_0 = -\,I_{sc}$, where I_{sc} is the short-circuit current flowing out of terminal a, which is the same as the Norton current I_N, i.e.,

$$D_0 = -\,I_N \cdots\cdots\cdots\cdots (2)$$

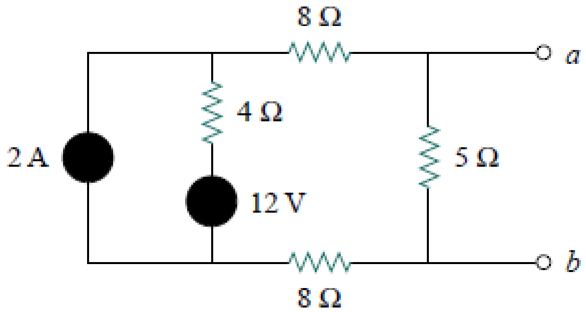
When all the internal independent sources are turned off, $D_0 = 0$ and the circuit can be replaced by an equivalent resistance R_{eq} (or an equivalent conductance G_{eq}= 1/Req), which is the same as R_{Th} or R_N. Thus equation (2) becomes

$$i = \frac{v}{R_{Th}} \text{ ……….. (3)}$$

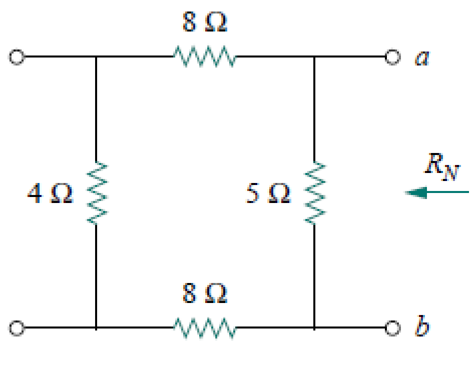
This expresses the voltage-current relation at terminals a-b of the circuit in figure (b), confirming that the two circuits in figure (a) and (b) are equivalent.

Examples:

1. Find the Norton equivalent circuit of the circuit in figure below.



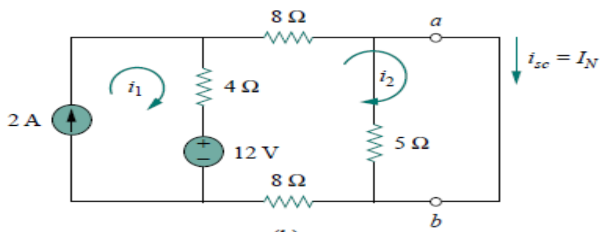
Ans. We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in figure (a), from which we find R_N. Thus,



(a)

$R_N = 5 \parallel (8+4+8) = 5 \parallel 20 = \frac{5\times20}{5+20} = 4\ \Omega$

To find I_N , we short-circuit terminals a and b, as shown in figure (b). We ignore the 5 Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain



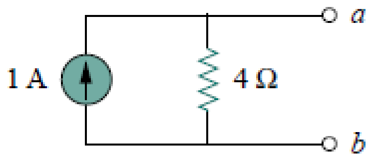
(b)

For mesh (1), i_1 = 2 A and for mesh (2), 4×(i_2-i_1) + 8i_2 + 5i_2 + 8i_2 -12 = 0 → 20 i_2 - 4 i_1 -12 = 0

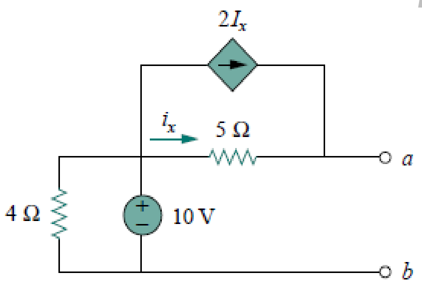
From these equations, we obtain

$$i_2 = 1 \text{ A} = i_{sc} = I_N$$

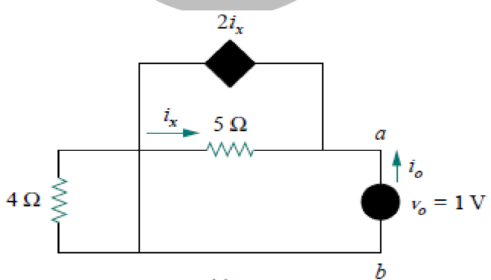
Thus, the Norton equivalent circuit is as shown in figure below.



2. Using Norton's theorem, find R_N and I_N of the circuit in figure at terminals a-b.



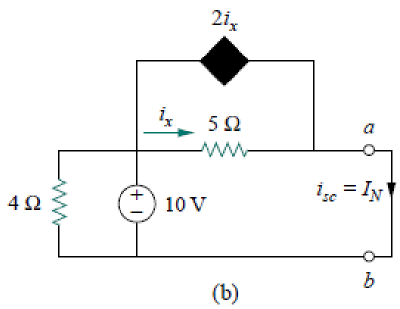
To find R_N, we set the independent voltage source equal to zero and connect a voltage source of v_0 = 1 V (or any unspecified voltage v_0) to the terminals. We obtain the circuit in figure (a). We ignore the 4Ω resistor because it is short-circuited. Also due to the short circuit, the5Ω resistor, the voltage source, and the dependent current source are all in parallel.



(a)

Hence, i_x = $\frac{v_0}{5}$ = 1/5 = 0.2 A. At node a, -i_0 = i_x+2i_x = 3 i_x = 0.6 A, and R_N = $\frac{v_0}{i_0}$ = -1.67 Ω

To find R_N, we short-circuit terminals a and b and find the current i_{sc}, as indicated in figure (b).



(b)

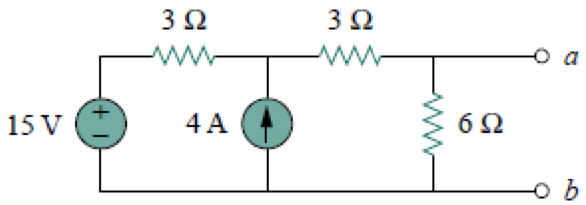
Note from this figure that the 4Ω resistor, the 10-V voltage source, the 5Ω resistor, and the dependent current source are all in parallel. Hence,

$$i_x = \frac{10-0}{5} = 2 \text{ A.}$$

At node a, KCL gives, $i_{sc} = i_x + 2i_x = 6 \text{ A} = I_N$

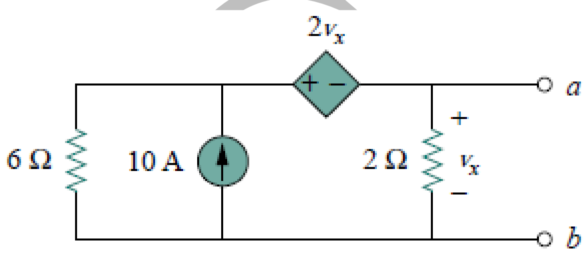
Problems:

1. Find the Norton equivalent circuit for the circuit in figure below.



Ans. R_N= 3Ω, I_N = 4.5 A.

2. Find the Norton equivalent circuit of the circuit in figure below.

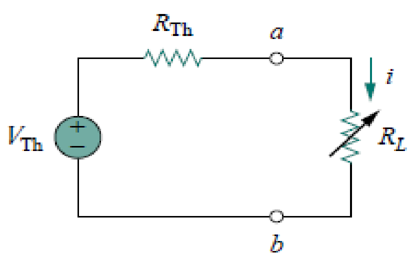


Ans. R_N= 1Ω, I_N = 10 A.

Maximum power transfer theorem:

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses.

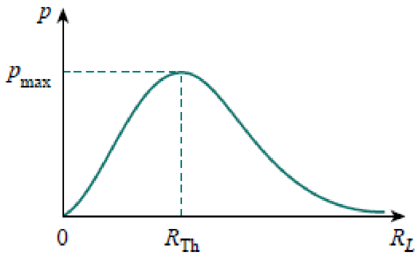
The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in figure below,



The power delivered to the load is,

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L \cdots\cdots\cdots\cdots (1)$$

For a given circuit, V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load varies as sketched in figure below.



We notice from figure that the power is small for small or large values of R_L but maximum for some value of R_L between 0 and∞.

Statement:

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load (R_L= R_{Th}).

Proof:

To prove the maximum power transfer theorem, we differentiate p in equation (1) with respect to R_L and set the result equal to zero. We obtain

$$\frac{dp}{dR_L} = {V_{Th}}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = 0 \rightarrow {V_{Th}}^2 \left[\frac{(R_{Th}+R_L) - 2R_L}{(R_{Th}+R_L)^3}\right] = 0 \rightarrow (R_{Th} + R_L) - 2R_L = 0$$

$$\boxed{R_L = R_{TH}}$$

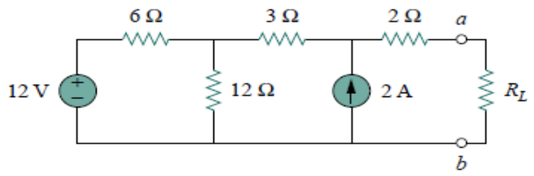
→ the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_L. We can readily show that $\left.\frac{d^2p}{d{R_L}^2}\right|_{R_L = R_{Th}} < 0.$

The maximum power transferred is obtained by substituting the value R_L= R_{Th} in equation (1) we get,

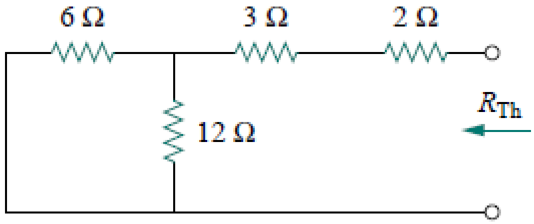
$$p_{max} = \frac{{V_{Th}}^2}{4R_{Th}}$$

<u>Example:</u>

Find the value of R_L for maximum power transfer in the circuit of figure below. Find the maximum power.



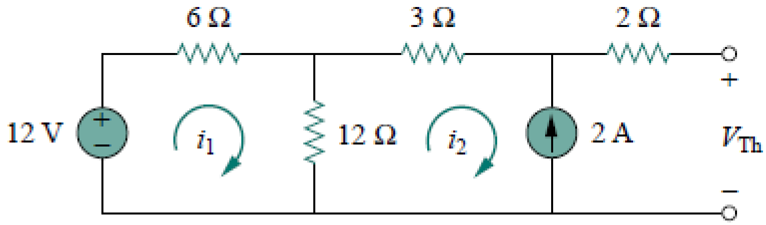
Ans. We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals a-b. To get R_{Th} , we use the circuit in figure (a) and obtain



(a)

R_{Th} = (2+3) + 6 || 12 = 5+$\frac{6\times12}{6+12}$ = 5 + 4 = 9 Ω

To get V_{Th} , we consider the circuit in figure (b).



(b)

Applying mesh analysis,

For mesh (1),

6 i_1+ 12(i_1- i_2) – 12 = 0 ⟶ 18 i_1- 12 i_2-12 =0

For mesh (2),

i_2 = - 2 A

Solving for i_1, we get i_1 =– 2/3 A. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

-12+ 6 i_1 + 3 i_2 + 2×0 + V_{Th}= 0 ⟶ V_{Th}= 22 V

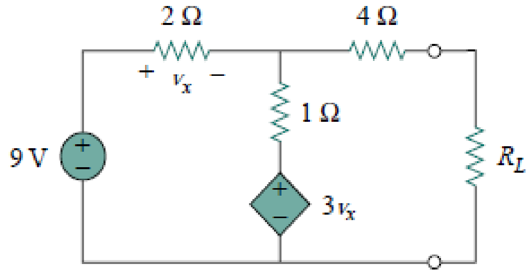
For maximum power transfer,

R_L= R_{Th} = 9 Ω

$p_{max} = \frac{{V_{Th}}^2}{4R_{Th}} = \frac{22^2}{4\times9}$ = 13.44 W

Problem:

Determine the value of R_L that will draw the maximum power from the rest of the circuit in figure below. Calculate the maximum power.

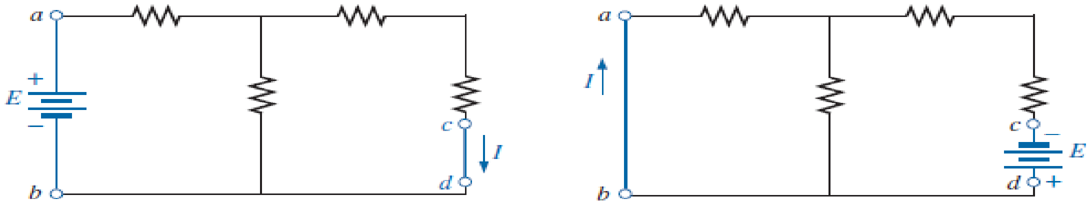


Reciprocity Theorem:

The reciprocity theorem is applicable only to single-source networks. It is, therefore, not a theorem used in the analysis of multisource networks described thus far.

Statement:

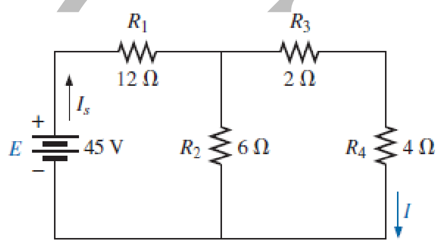
The current I in any branch of a network due to a single voltage source E anywhere else in the network will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.



(a) (b)

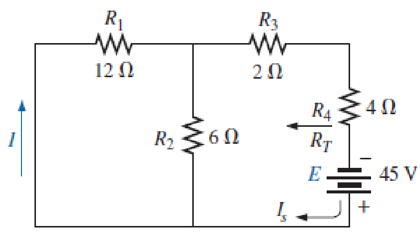
Proof:

To demonstrate the validity of this statement and the theorem, consider the network in above figure, in which values for the elements of figure below have been assigned. The total resistance is



$R_T = R_1 + R_2 \parallel (R_3 + R_4) = 12 + 6 \parallel (2+4) = 12 + \frac{6\times6}{6+6} = 12 + 3 = 15\ \Omega$ and $I_s = \frac{E}{R_T} = \frac{45}{15} = 3$ A

Therefore, $I = \frac{I_s}{2} = \frac{3}{2} = 1.5$ A



For the network in figure above, which corresponds to that in figure (b), we find

$R_T = R_4 + R_3 + R_2 \parallel R_1 = 4 + 2 + 6 \parallel 12 = 6 + \frac{6\times12}{6+12} = 6 + 4 = 10\ \Omega$ and $I_s = \frac{E}{R_T} = \frac{45}{10} = 4.5$ A

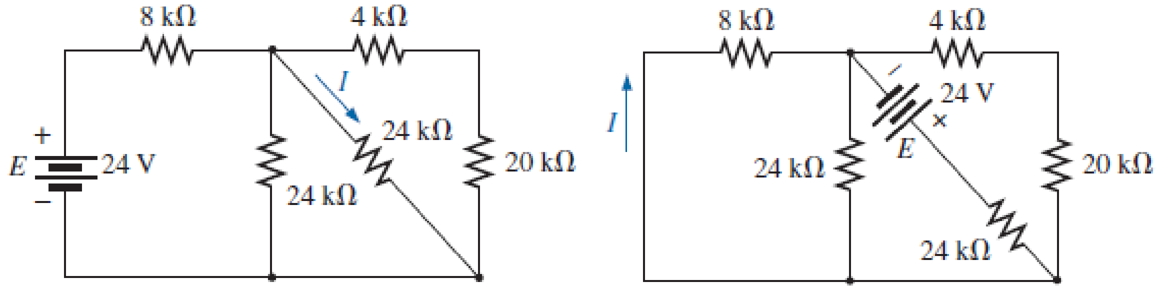
Therefore, $I = \frac{6\Omega\times4.5A}{12\Omega+6\Omega} = \frac{4.5A}{3} = 1.5$ A →agrees with the above result.

Problem:

i) For the network in figure (a), determine the current I.

ii) Repeat part (i) for the network in figure (b).

iii) Is the reciprocity theorem satisfied?



(a) (b)