the isotope ^{235}U has a concentration less than its natural value. Although depleted uranium is referred to as a by-product of the enrichment process, it does have uses in the nuclear field and in commercial and defense industries.

2.4. Mass Defect and Binding Energy

The separate laws of conservation of mass and conservation of energy are not applied strictly to the nuclear level. It is possible to convert between mass and energy. Instead of two separate conservation laws, a single conservation law states that the sum of mass and energy is conserved. Mass does not magically appear and disappear at random. A decrease in mass will be accompanied by a corresponding increase in energy and vice versa.

Before going on the discussion of mass defect and binding energy, it is convenient to introduce a conversion factor derived by the mass-energy relationship from Einstein's Theory of Relativity.

Einstein's famous equation relating mass and energy is $E = mc^2$ (Eq.1.1) where c is the velocity of light ($c = 2.998 \times 10^8$ m/sec). The energy equivalent of 1 u can be determined by inserting this quantity of mass into Einstein's equation and applying conversion factors.

$$\begin{aligned} E &= mc^2 \\ &= 1\text{u}\{1.6606\times10^{-27}\text{kg/1u}\}\,\{2.998\times10^{8}\text{m/sec}\}^2 \\ &\qquad (1\text{N}/1\text{kg.m/sec}^2)(1\text{J}/1\text{N.m}) \\ &= 1.4924\times10^{-10}\,\text{J}\{1\ \text{MeV}/1.6022\times10^{-13}\ \text{J}\} \\ &= 931.5\ \text{MeV} \end{aligned}$$

2.4.1. Mass defect

Careful measurements have shown that the mass of a particular atom or isotope is always slightly less than the number of nucleons (sum of the individual neutrons and protons) of which the atom consists. The difference between the atomic mass of the atom and the total number of nucleons (A) in the nucleus is called the *mass defect or mass excess* (Δm). The mass defect can be expressed in terms of atomic mass units and/or in terms of energy as:

$$\Delta m = M(Z,N) - A \text{ u} \qquad 2.2a$$

$$\Delta m = \{M(Z,N) - A\}931.5\, MeV \qquad 2.2b$$

where: Δm = mass defect (u or MeV)
M (Z, N) = mass of nuclide ${}^{A}_{Z}X$ (u)
A = mass number

In calculating the mass defect, it is important to use the full accuracy of mass measurements because the difference in mass is small compared to the mass of the atom. Rounding off the masses of atoms and particles to three or four significant digits prior to the calculation will result in a calculated mass defect of zero.

Example:

Calculate the mass defect for lithium-7. The mass of ^{7}Li is 7.016003 u.

Solution:

Apply Eq. 2.2a
Δm = 7.016003 -7
= 0.016003 u

Applying Eq. 2.2b
Δm = (7.016003 -7)931.5
= 14.9067945 MeV

2.4.2. Binding energy

Binding energy is defined as the amount of energy that must be supplied to a nucleus to completely separate its nuclear particles (nucleons). It can also be understood as the amount of energy that would be released if the nucleus was formed from the separate particles.

Since 1 u is equivalent to 931.5 MeV of energy, the binding energy can be calculated by the mass difference between the nucleus and the sum of those of the free nucleons, including the mass of electrons associated with protons.

$$BE(Z,A) = \{Zm_p + Zm_e + (A-Z)m_n - M(Z,N)\}c^2 \quad 2.3a$$

or

$$BE(Z,A) = \{Zm_H + Nm_n - M(Z,N)\}931.5\,MeV \quad 2.3b$$

$m_p =$ mass of proton (1.0072764 u)
$m_n =$ mass of neutron (1.008665 u)
$m_e =$ mass of electron (0.000548597 u)
$m_H = m_p + m_e =$ mass of hydrogen atom = (1.007825 u)

Appendix II tabulates, in addition to some other usefull informations, the atomic weight of all elements and the mass defect in MeV of all important isotopes. These values can be used to find the atomic mass of each isotope.

Example:

Calculate the mass defect and binding energy for uranium-235. One uranium-235 atom has a mass of 235.043924 u.

Solution:

Step 1: Calculate the mass defect using Equation (2.2)

$$\Delta m = \{M(Z,N)-A\}931.5\ MeV$$
$$= (235.043924 - 235)931.5 = 40.9152\ \text{MeV}$$

Step 2: Use the mass defect and Equation (2.3) to calculate the binding energy.

$$BE = \{Zm_H + Nm_n - M(Z,N)\}931.5\ MeV$$
$$= \{[92(1.007826\ \text{u}) + (235\text{-}92)1.008665\ \text{u}] - 235.043924\}931.5$$
$$BE = 1.91517\ \text{u} \times 931.5\ \text{MeV/u} = 1784\ \text{MeV}$$

2.4.3. Separation energy

The useful and interesting property of the binding energy is the neutron and proton separation energies. The neutron separation energy S_n *is the amount of energy required to remove a neutron from a nucleus,* ${}^{A}_{Z}X$ (sometimes called the binding energy of the last neutron). This is equal to the difference in binding energies between ${}^{A}_{Z}X$ and ${}^{A-1}_{Z}X$.

$$S_n = BE\,({}^{A}_{Z}X) - BE\,({}^{A-1}_{Z}X) \qquad 2.4$$

$$S_n = \{M\,({}^{A-1}_{Z}X) - M\,({}^{A}_{Z}X) + m_n\}\,c^2 \qquad 2.5$$

Similarly one can define proton separation energy S_p *as the energy needed to remove a proton from a nucleus* ${}^{A}_{Z}X$ (also called the binding energy of the last proton) which convert to another nuclide, ${}^{A-1}_{Z-1}Y$ and can be calculated as follows.

$$S_p = BE\,({}^{A}_{Z}X) - BE\,({}^{A-1}_{Z-1}Y) \qquad 2.6$$

$$S_p = \{M\,({}^{A-1}_{Z-1}Y) - M\,({}^{A}_{Z}X) + m\,({}^{1}H)\}\,c^2 \qquad 2.7$$

The Hydrogen mass appears in this equation instead of proton mass, since the atomic mass is m $(^{1}H) = m_p + m_e$. The neutron and proton separation energies are analogous to the ionization energies in atomic physics, in terms of the binding of the outermost valance nucleons. Just like the atomic ionization energies, the separation energies show evidence for nuclear shell structure that is similar to atomic shell structure.

2.4.4. Binding energy per nucleon

As with many other nuclear properties that will be discussed in the coming sections, we gain valuable clues to nuclear structure from a systematic study of nuclear binding energy. As the number of particles in a nucleus increases, the total binding energy also increases. The rate of increase, however, is not uniform. This lack of uniformity results in a variation in the amount of binding energy associated with each nucleon within the nucleus. In other wards since the binding energy increases more or less linearly with atomic mass number A, it is convenient to show the variation of the average binding energy per nucleon, BE/A as function of A. Fig. 2.4 shows the average BE/A as plotted versus atomic mass number A.

Fig. 2.4 illustrates that as the atomic mass number increases, the binding energy per nucleon increases almost linearly for light elements (except for ${}^{4}_{2}He, {}^{8}_{4}Be, {}^{12}_{6}C, {}^{16}_{8}O$), then rapidly for A < 60 reaches a maximum value of 8.79 MeV at A = 56 (close to iron) and decreases to about 7.6 MeV for A = 238. The average BE/A of most nuclei is, to within 10%, about 8 MeV.

The general shape of the BE/A curve can be explained using the general properties of nuclear forces. Very short-range attractive nuclear forces that exist between nucleons hold the nucleus together. On the other hand, long-range repulsive electrostatic (coulomb) forces that exist between all the protons in the nucleus are forcing the nucleus apart. So the nuclear forces are very short range of the order of the diameter of one nucleon, or they saturate to pairs of nucleons (two protons and two neutrons) to form α-cluster. This is clear in light stable nuclei for A < 20 where the sharp rise appear to be off the curve. Those specific light stable nuclei are ${}^{4}_{2}He, {}^{8}_{4}Be, {}^{12}_{6}C, {}^{16}_{8}O$. This is due to the fact of higher BE/A of ${}^{4}_{2}He$ particle (or α-cluster) bounded in the nucleus and the other (A = 4n nuclei for n=1, 2 ...) stable nuclei for A < 20 compared to their neighbors. In other words, the 4n nuclei for n=1, 2, .. trend to make α-particles.

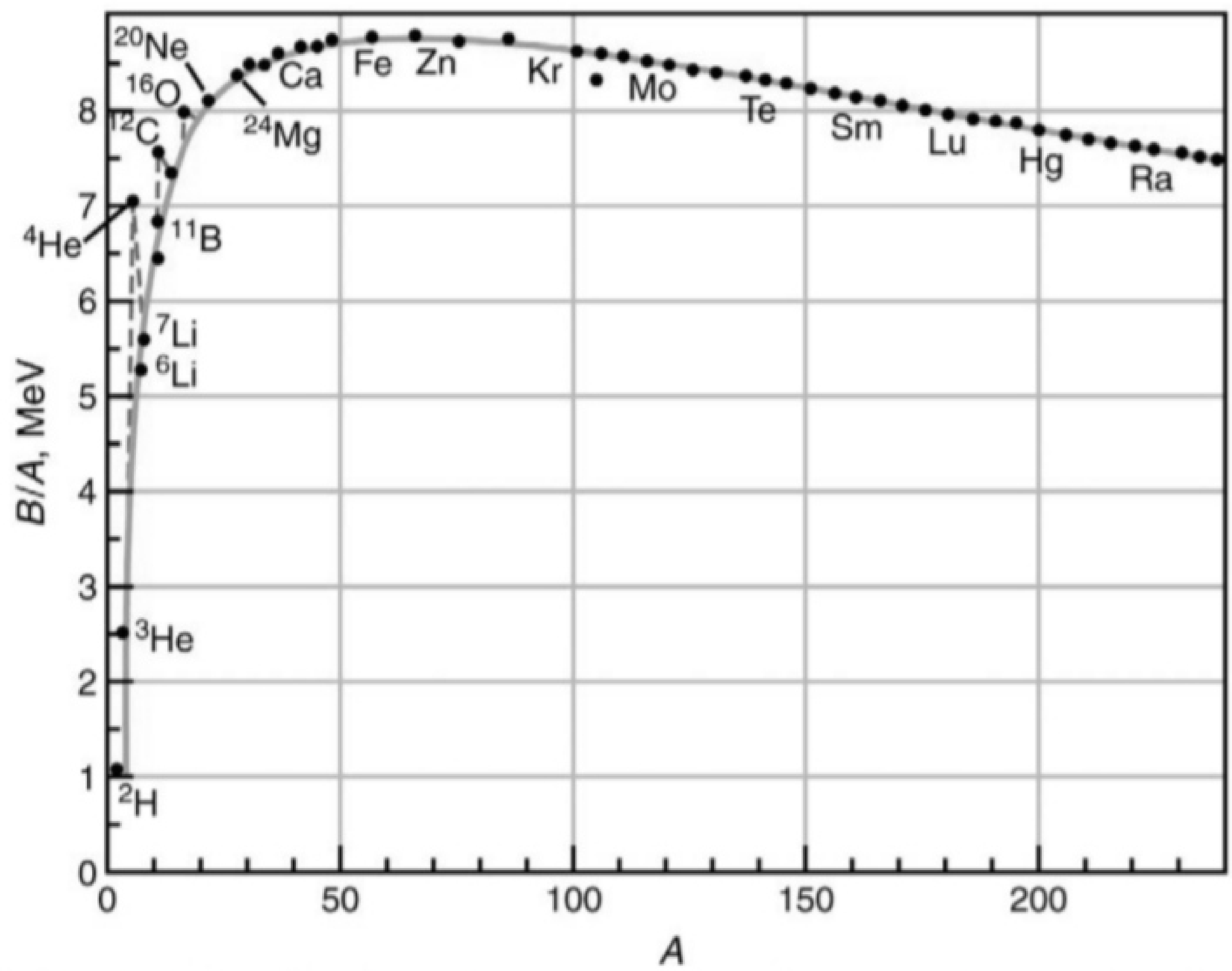

Figure 2.4. Binding Energy per Nucleon vs. Mass Number.

As the atomic number and the atomic mass number increase, the repulsive electrostatic forces within the nucleus increase due to the greater number of protons in the heavy elements. To overcome this increased repulsion, the proportion of neutrons in the nucleus must increase to maintain stability. This increase in the neutron-to-proton ratio only partially compensates for the growing proton-proton repulsive force in the heavier, naturally occurring elements. Because the repulsive forces are increasing, less energy must be supplied, on the average to remove a nucleon from the nucleus. The BE/A has decreased.

The BE/A of a nucleus is an indication of its degree of stability. Generally, the more stable nuclides have higher BE/A than the less stable ones. The increase in the BE/A as the atomic mass number decreases from 260 to 60 is the primary reason for the energy liberation in the fission process. In addition, the increase in the BE/A as the atomic mass number increases from 1 to 60 is the reason for the energy liberation in the fusion process, which is the opposite reaction of fission.

The heaviest nuclei require only a small distortion from a spherical shape (small energy addition) for the relatively large coulomb forces forcing the two halves of the nucleus apart to overcome the attractive nuclear forces holding the two halves together. Consequently, the heaviest nuclei are easily fissionable compared to lighter nuclei.

2.5. Mass Spectroscopy

Binding energies may be calculated if masses are measured accurately. One way of doing this is by using the techniques of mass spectroscopy. The principle of the method is shown in Fig. 2.5.