PHY-H-CC-T-03: ELECTRICY AND MAGNETISM

LECTURE-8 (Pabitra Halder, Assistant Professor, Department of Physics, Berhampore Girls' College)

Electromagnetic Induction

Definition:

Whenever there is a change of flux through a closed circuit an electro motive force (e.m.f) and hence a current is induced in the circuit. This phenomenon is called electromagnetic induction. The result of Faraday led to development of two useful laws:

1. Neumann's law:

The induced electromotive force (e.m.f) in a circuit is equal to the rate of change of magnetic flux linked with the circuit.

2. Lenz's law:

The direction of induced e.m.f is decided by Lenz's law which states that in all cases of electromagnetic induction the direction of the induced e.m.f will be such to oppose the very cause to which it is due to.

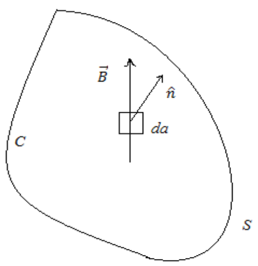
If Φ is the flux linked with a circuit at a time t, and then $\frac{d\Phi}{dt}$ gives the rate of variation of flux. The combination of the two laws of electromagnetic induction yields

$\mathcal{E} = -\frac{d\Phi}{dt}$ → integral form of Faraday's law of electromagnetic induction

Where $\mathcal{E}$ is the induced emf. The negative sign signifies that the e.m.f oppose the change of flux. Hence the induced e.m.f is often referred to as the back emf.

The SI unit of $\mathcal{E}$ is expressed in volt.

Differential form of Faraday's law:



Let C be a closed circuit binding an open surface S, placed in a magnetic field $\vec{B}$. The flux Φ through S is given by

$$\Phi = \iint \vec{B}.\hat{n}\, da \quad \cdots\cdots\cdots\cdots (1)$$

The induced e.m.f generated around C is given by $\mathcal{E} = -\frac{d\Phi}{dt}$ ············ (2)

If the electric field is $\vec{E}$, the induced e.m.f around the curve C, is given by

$$\mathcal{E} = \oint \vec{E}.\vec{dl} \quad \cdots\cdots\cdots\cdots (3)$$

With the aid of equations (1) and (3), equation (2) can be written as

$$\oint \vec{E}.\vec{dl} = -\frac{d}{dt}\left(\iint \vec{B}.\hat{n}\, da\right) \cdots\cdots\cdots\cdots (4)$$

Using Stokes' theorem we get

$$\iint (\vec{\nabla} \times \vec{E}).\hat{n}\, da = -\iint \frac{\partial \vec{B}}{\partial t}.\hat{n}\, da \cdots\cdots\cdots\cdots (5)$$

The above equation true for any arbitrary fixed surface S. Therefore,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow \textbf{differential form of Faraday's law}$$

Self-induction:

We consider a circuit carrying a varying current. Let the current at any instant t be I (emu), which is changing at a rate $\frac{dI}{dt}$ emu/sec.

The instantaneous flux linking the coil will be directly proportional to the instantaneous current I

$\Phi \propto I \rightarrow \Phi = L\, I \rightarrow \frac{d\Phi}{dt} = L\frac{dI}{dt} \rightarrow |\mathcal{E}| = L\frac{dI}{dt}$

The co-efficient L is called the co-efficient of self-induction or self-induction of the coil.

If I= 1 emu then Φ = L. Again, if $\frac{dI}{dt}$ = 1 emu/sec, then $|\mathcal{E}|$ = L.

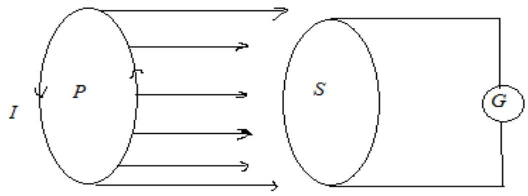
Definition:

Thus the self-induction of a coil is defined as the flux linked with the coil when it carries unit current. It can also be defined as the magnitude of the induced emf produced in the coil when the rate of change of current through it is maintained unity.

Unit:

Its practical unit is Henry. 1 Henry = 10^9 emu

Mutual induction:



We consider two coils P and S placed closed to each other. If the varying current at any time t, I amp passed through the P- coil and amount of flux emitted from it will be linked up with the S-coil. Evidently, the flux linked up with the S-coil will be proportional to the current flowing through the P-coil. If Φ be the flux linked with the S-coil when the current in the P-coil is I amp, we may write

$\Phi \propto I \rightarrow \Phi = M\, I \rightarrow \frac{d\Phi}{dt} = M\frac{dI}{dt} \rightarrow |\mathcal{E}| = M\frac{dI}{dt}$

Where M is a constant which depends on the pair of coil. The co-efficient M is called the co-efficient of mutual-induction.

Definition:

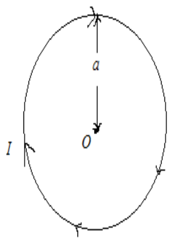
The mutual-induction or the co-efficient of mutual induction is defined as the amount of flux linked with the secondary coil when the current flowing through the primary coil is unity.

It can also define as the magnitude of the induced emf in the secondary coil when the rate of change of current in the primary coil is maintaining unity.

Calculation of self-induction:

1. A circular coil:

Let a circular coil of radius a having n turns of wire wound over it, carry a varying current whose value at a time t is *I* ampere.



The magnetic induction at the centre O of the coil due to this current *I* is $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\pi n I}{a}$ wb/m^2. Where μ_0 is the permeability of free space.

Hence, the flux linking each turn of the coil at a time t is

$\Phi_1 = \iint \vec{B}.\hat{n}\, da = \frac{\mu_0}{4\pi}\frac{2\pi n I}{a} \times \pi a^2 = \frac{\mu_0}{2}\pi n a I$ wb.

Hence, the instantaneous total flux linked with n turns of the coil is

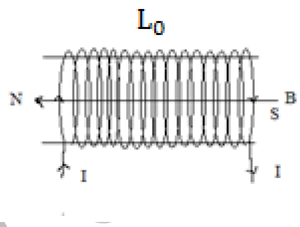
$\Phi = n\,\Phi_1 = \frac{\mu_0}{2}\pi n^2 a I$ wb.

Therefore, $|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{\mu_0}{2}\pi n^2 a \frac{dI}{dt}$

If $\frac{dI}{dt} = 1$ amp/sec; $|\mathcal{E}| = L = \frac{\mu_0}{2}\pi n^2 a$ Henry $= \frac{\mu_0}{2}\pi n^2 a \times 10^9$ emu.

2. A long Solenoid:

We consider a air-cored long solenoid having N turns of wire and of length L_0. The solenoid carries a current whose instantaneous value is *I* ampere.



The magnetic induction or magnetic flux density at any point P inside a solenoid at a time t is

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{4\pi N I}{L_0}$$

If α be the area of cross-section of the solenoid, the flux linked with each turn of the solenoid at time t is $\Phi_1 = \iint \vec{B}.\hat{n}\, da = \frac{\mu_0}{4\pi}\frac{4\pi N I}{L_0} \times \alpha = \frac{\mu_0 N I \alpha}{L_0}$

Therefore, the total flux linking the entire solenoid is

$\Phi = N\,\Phi_1 = \frac{\mu_0 N^2 I \alpha}{L_0}$

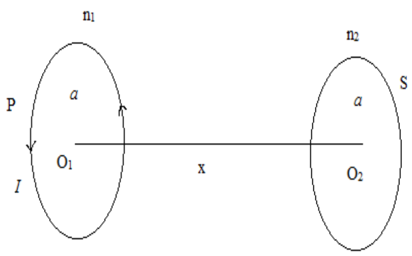
Therefore, $|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{\mu_0 N^2 \alpha}{L_0}\frac{dI}{dt}$

If $\frac{dI}{dt} = 1$ amp/sec; $|\mathcal{E}| = L = \frac{\mu_0 N^2 \alpha}{L_0}$ Henry $= \frac{\mu_0 N^2 \alpha}{L_0} \times 10^9$ emu.

Calculation of mutual-induction:

1. Pairs of circular coil:

We consider a pair of co-axial circular coils of radius a. One of them (P-coil) has n_1 turns of wire, while the other (S-coil) has n_2 turns. The P-coil carries a varying current whose value at time t is *I* A.



Now the magnetic induction at the centre O_2 at the S-coil due to P-coil is
$\vec{B} = \frac{\mu_0 n_1 a^2 I}{2(a^2+x^2)^{3/2}}$ .... (1), where x is the distance between the centres of two coils.

Hence, the flux linking each turn of the S-coil at time t is $\Phi_1 = \frac{\mu_0 n_1 a^2 I}{2(a^2+x^2)^{3/2}} \times \pi a^2$ .... (2)

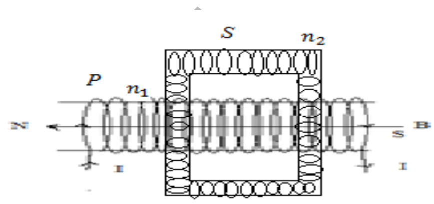
Thus, the total flux linking the S-coil at time t is

$\Phi = n_2 \Phi_1 = \frac{\mu_0 n_1 n_2 \pi a^4}{2(a^2+x^2)^{3/2}} I$

Therefore, $|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{\mu_0 n_1 n_2 \pi a^4}{2(a^2+x^2)^{3/2}} \frac{dI}{dt}$

If $\frac{dI}{dt} = 1$ amp/sec; $|\mathcal{E}| = M = \frac{\mu_0 n_1 n_2 \pi a^4}{2(a^2+x^2)^{3/2}}$ Henry $= \frac{\mu_0 n_1 n_2 \pi a^4}{2(a^2+x^2)^{3/2}} \times 10^9$ emu.

2. Solenoid:



We consider a long solenoid whose primary coil P and secondary coil is S. The P-coil has n_1 turns of wire wound over it, while the total turns on the S-coil are n_2. The coil P and S are kept insulated from each other.

When the P-coil carries a varying current whose instantaneous value is *I* ampere an induced back emf will be produced in the S-coil. The magnetic induction or flux density at any axial point will inside the primary P is $\vec{B} = \frac{\mu_0 n_1 I}{l}$; where μ_0 is the permeability of the core medium and l be the length of the P-coil.

If A be the area of cross-section of the P-coil that of S-coil approximately A. Hence, the flux linking each turn of the S-coil is $\Phi_1 = \frac{\mu_0 n_1 I}{l} \times A$.

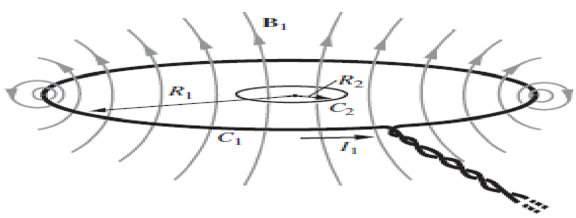
Thus, the total flux linked with the S-coil at a time t is

$\Phi = n_2 \Phi_1 = \frac{\mu_0 n_{1} n_2 A}{l} I$

Therefore, $|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{\mu_0 n_{1} n_2 A}{l} \frac{dI}{dt}$

If $\frac{dI}{dt} = 1$ amp/sec; $|\mathcal{E}| = M = \frac{\mu_0 n_{1} n_2 A}{l}$ Henry $= \frac{\mu_0 n_{1} n_2 A}{l} \times 10^9$ emu.

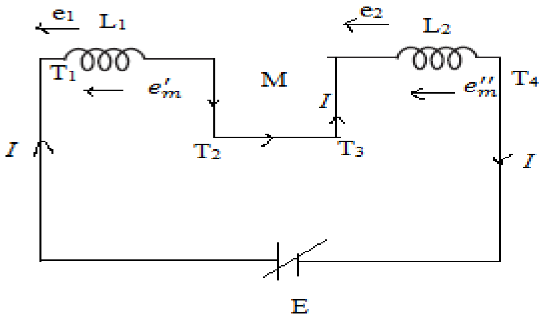
A reciprocity theorem:



If the coefficient of mutual-inductance (M_{12}) is the (negative) ratio of the electromotive force in circuit C_1, caused by changing current in circuit C_2 , to the rate of change of current I_2 and that for circuit C_2 is M_{21} then for any two circuit M_{12} = M_{21}.This is a reciprocity theorem.

Calculation of effective inductances:

1. Series combination:



When two coil of self inductances L_1 and L_2 are connected in series, the flux lines emitted by the two coils will be in the same direction.

Let I be the instantaneous current flowing through each coil at time t. If $\frac{dI}{dt}$ be the rate of change of current, the induced back emf due to self-induction in the first coil

$$e_1 = -L_1\frac{dI}{dt} \cdots\cdots\cdots\cdots (1)$$

The mutual induced emf in the first coil due to the second coil will be $e'_m = -M\frac{dI}{dt} \cdots\cdots\cdots\cdots (2)$

Similarly, the corresponding quantities for the second coil are $e_2 = -L_2\frac{dI}{dt} \cdots\cdots\cdots\cdots (3)$ and

$e''_m = -M\frac{dI}{dt} \cdots\cdots\cdots\cdots (4)$

The direction of all induced emfs will be in opposite with the applied emf E.

The resultant emf of the combination is $e = -(L_1+L_2+2M)\frac{dI}{dt} \cdots\cdots\cdots\cdots (5)$

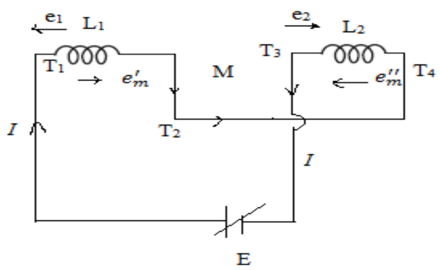
If L_{eq} be the equivalent self inductance of the combination, we must have,

$e = -L_{eq}\frac{dI}{dt} \cdots\cdots\cdots\cdots (6)$

Comparing (5) and (6); we have

$$L_{eq} = L_1+L_2+2M$$

2. Series in opposition:



In this case the induced emfs e_1 and e_2 due to self-induction in the two coils oppose the applied emf. But the mutual induced emfs in the two coils will be the same direction as that of applied emf.

The emfs are given by

$e_1 = -L_1\frac{dI}{dt}$; $e_2 = -L_2\frac{dI}{dt}$ and $e'_m = M\frac{dI}{dt}$; $e''_m = M\frac{dI}{dt}$

Therefore, the resultant induced emf in this case will be e = - (L_1+L_2-2M) $\frac{dI}{dt}$ ………… (1)

If L'_{eq} be the equivalent self-inductance of the combination, we have

e = - L'_{eq} $\frac{dI}{dt}$ ………… (2)

Comparing (1) and (2), we get

$$L'_{eq} = L_1 + L_2 - 2M$$