CHAPTER 4

RADIOACTIVE DECAY DYNAMIC

The rate at which a sample of radioactive materia decays is not constant. As individual atoms of the materia decay, there are fewer of those types of atoms remaining Since the rate of decay is directly proportional to th number of atoms, the rate of decay will decrease as th number of atoms decreases.

4.1. Radioactive Decay Rates

Radioactivity is the property of certain nuclides o spontaneously emitting particles or electromagnetic waves or is the process in which an unstable atomic nucleus lose energy by emitting radiation in the form of particles o gamma radiation. This decay, or loss of energy, results in a atom of one type, called the parent nuclide transforming t an atom of a different type, called the daughter nuclide. Thi is a random process on the atomic level, in that it i impossible to predict when a particular atom will decay However, the average behavior of a very large sample ca be predicted accurately by using statistical methods. Thes studies have revealed that there is a certain probability that in a given time interval, a certain fraction of the nucle within a sample of a particular nuclide will decay. Thi probability per unit time that an atom of a nuclide will deca is known as the *radioactive decay constant*, λ. The units fo the decay constant are inverse time.

4.2. Units of Radioactivity

The *activity* (A) of a sample of any radioactive nuclide is the rate of decay of the nuclei of that sample. For a sample containing billions of atoms, this rate of decay is usually measured in the number of disintegrations that occur per second. If N is the number of nuclei present in the sample at a certain time, the change in number of those nuclei with time, rate of decay, is the activity A, and can be given by:

$$A = -\frac{dN}{dt} \qquad 4.1$$

The minus sign is used to make A a positive quantity since dN/dt is, of course, intrinsically negative.

In addition, the activity is the product of the decay constant and the number of atoms present in the sample. The relationship between the activity A, number of atoms N, and decay constant λ is given by:

$$A = \lambda N \qquad 4.2$$

Since λ is a constant, the activity and the number of atoms are always proportional.

Two common units to measure the activity of a substance are the Curie (Ci) and Becquerel (Bq). A *curie* is a unit of measure of the rate of radioactive decay equal to 3.7 x 10^{10} disintegrations per second. This is approximately equivalent to the number of disintegrations that one gram of radium ^{226}Ra will undergo in one second. A *Becquerel,* as the metric system, is a more fundamental unit of measure of radioactivity than Curie. The conversion between Curie and Becquerel is shown below.

1 Bq = 1 dis/sec

1 Curie = 3.7×10^{10} Bq

Note that the activity tells us only the number of disintegrations per second; it says nothing about the kind if radiations emitted their energies, or the effect of radiation on a biological system, since different radiations may give different effects. In the next section, some alternative units for measuring radiation that take into account their relative biological effects will be discussed.

4.3. Radioactive Decay Law

From the previous two basic relationships, Eq.4.1 and Eq. 4.2, it is possible to use calculus to derive an expression that can be used to calculate how the number of atoms present will change over time.

$$- dN = N\lambda\, dt$$

or

$$-\frac{dN}{N} = \lambda dt \qquad 4.3$$

This equation describes the situation for any short time interval, *dt*. To find out what happens for all periods of time, we simply add up what happens in each short time interval. In other words, we integrate the above equation. Expressing this more formally, we can say that for the period of time from $t = 0$ to any later time t, the number of radioactive nuclei will decrease from N_0 to N_t, so that:

$$N_t = N_0\, exp\,(-\lambda t) \qquad 4.4$$

This final expression is known as the *Radioactive Decay Law*. It tells us that the number of radioactive nuclei N_0 at time $t = 0$ will decrease in an exponential fashion to N_t with time with the rate of decrease being controlled by the decay probability per unit time, decay constant λ.

The decay constant is characteristic of individual radionuclide, i.e., has a different values for each. Some, like uranium-238, have a small value and the material, therefore, decays quite slowly over a long period of time. Other nuclei such as technetium-99m ($^{99}Tc^{*}$) are metastable, have a relatively large decay constant and decay far more quickly. The radioactive decay law is shown in graphical form in Fig. 4.1 for three typical radionuclides of different decay constants.

The graph plots the number of radioactive nuclei at any time, N_t against time, t. The influence of the decay constant can be seen clearly.

Since the activity A and the number of atoms N are always proportional, they may be used interchangeably to describe any given radionuclide population. Therefore, the following is true:

$$A_t = A_0 \exp(-\lambda t) \qquad 4.5$$

where:

A_t = activity present at time t
A_0 = activity initially present

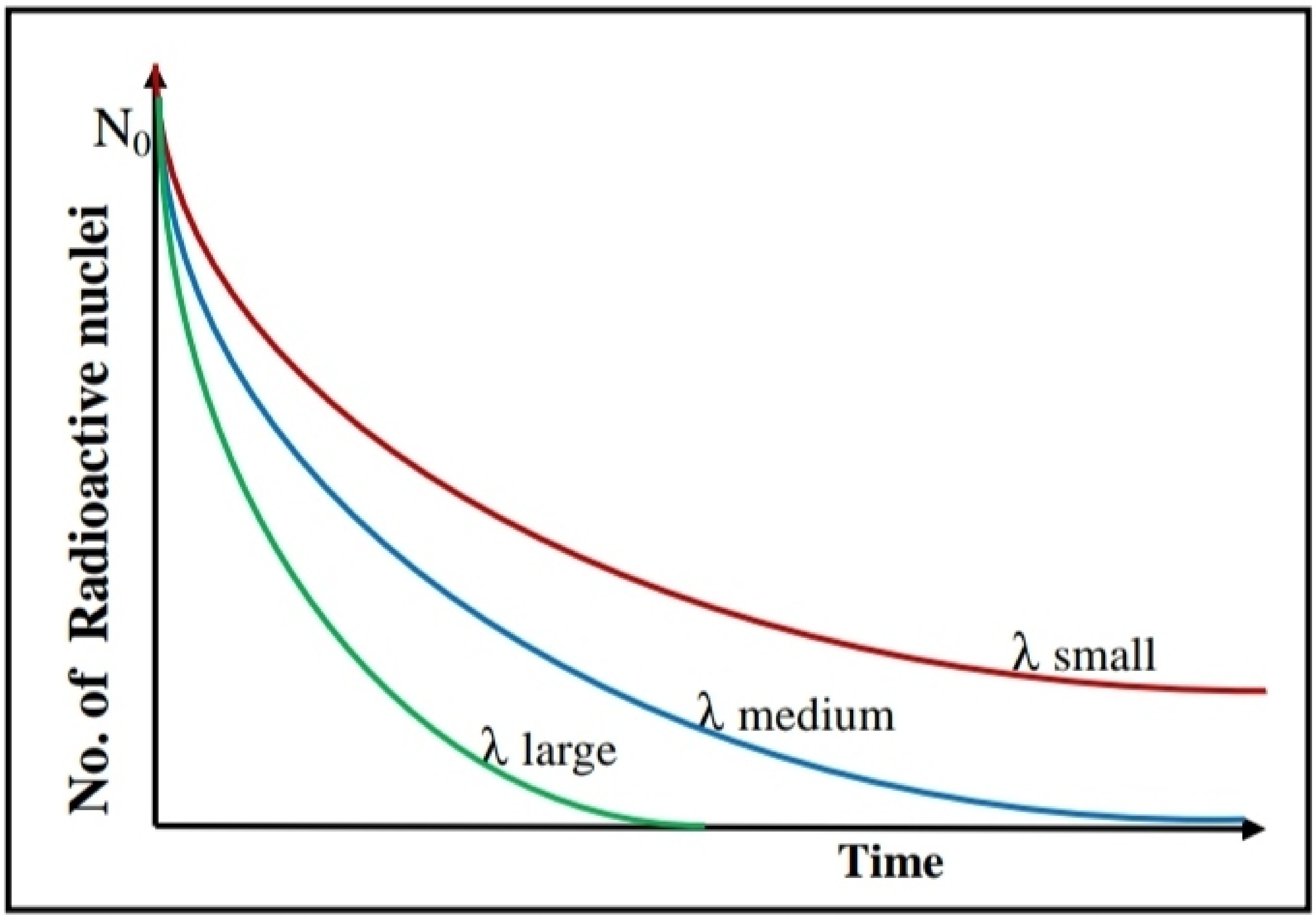

Figure 4.1. The influence of the decay constant.

4.4. Radioactive Half-Life

One of the most useful terms for estimating how quickly a nuclide will decay is the radioactive half-life. The *radioactive half-life* is defined as the amount of time required for the activity to decrease to one-half of its original value. A relationship between the half-life and decay constant can be developed from Eq.4.5. The half-life can be calculated by solving Eq.4.5 for the time, t, when the current activity, *A*, equals one-half the initial activity $A = \frac{1}{2}A_0$. First, solve Eq.4.5 for *t*:

$$t = -\frac{Ln(A_t / A_0)}{\lambda} \qquad 4.6$$

Now if A is equal to one-half of A_0, A/A_0 is equal to one-half. Substituting this ratio in the above equation yields an expression for $t_{½}$:

$$t_{1/2} = -\frac{Ln(1/2)}{\lambda} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} \qquad 4.7$$

The basic features of decay of a radionuclide sample are shown by the normalized graph in Fig. 4.2.

Note that the half-life does not express how long a material will remain radioactive but simply the length of time for its radioactivity to reduce by half. Assuming an initial number of atoms N_o, the population, and consequently, the activity may be noted to decrease by one-half of this value in a time of one half-life. Additional decreases occur so that whenever one half-life elapses; the number of atoms drops to one-half of what its value was at the beginning of that time interval. After five half-lives have elapsed, only 1/32, or 3.1%, of the original number of atoms remains. After seven half-lives, only 1/128, or 0.78%, of the atoms remains. The number of atoms existing after 5 half-lives can usually be assumed to be negligible.

Another useful term is the *mean lifetime* of the nuclei, which is given by the total time of existence of all nuclei divided by the number of nuclei present initially. Since the decay process is a statistical one, any single atom may have a life from *zero* to ∞. Hence, the mean lifetime τ is given:

$$\tau = \frac{1}{N_0}\int_0^{\infty} N_0 \lambda t e^{-\lambda t} dt = \lambda \int_0^{\infty} t e^{-\lambda t} dt = \frac{1}{\lambda} \qquad 4.8$$

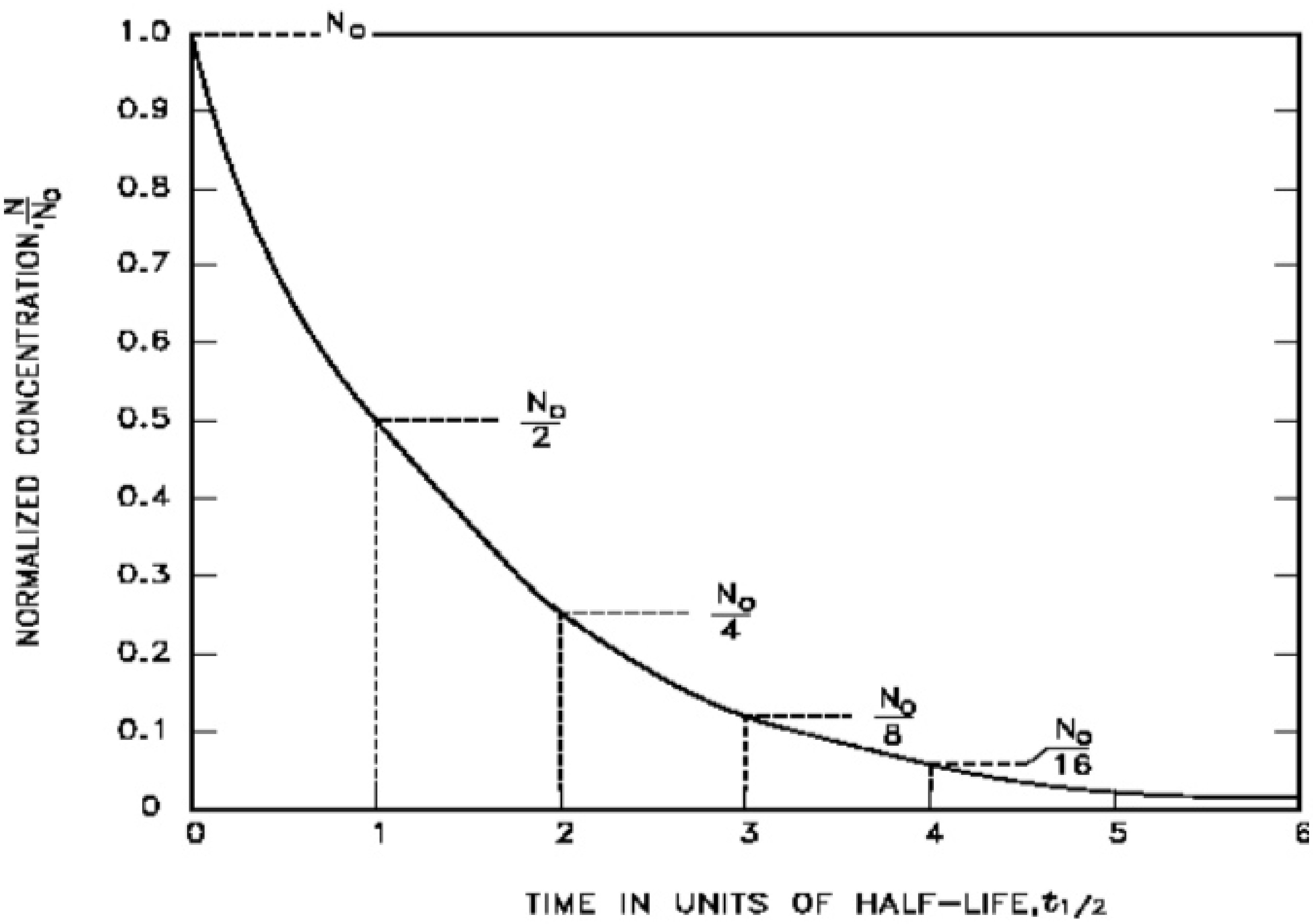

Figure 4.2. Radioactive decay as a function of time in units of half-life.

It is also possible to consider the radioactive decay law from another perspective by plotting the logarithm of N against time. In other words from our analysis above by plotting the expression:

$$Ln(N/N_0) =- \lambda t \quad \text{in the form} \quad Ln(N) = -\lambda t + Ln(N_0) \qquad 4.9$$

Notice that this expression is simply an equation of the form $y = mx + c$ where $m = -\lambda$ and $c = ln(N_0)$. As a result, it is the equation of a straight line of slope $-\lambda$ as shown in Fig. 4.3. Such a plot is sometimes useful when we wish to consider a situation without the complication of the direct exponential behavior.