$$-Q = \frac{1}{2} m_a \upsilon_a^2 \left(\frac{M_X}{M_X + m_a} \right) \quad 3.9$$

Then the threshold energy is then:

$$T_{th} = \frac{1}{2} m_a \upsilon_a^2 = (-Q)\left(1 + \frac{m_a}{M_X} \right) \quad 3.10$$

The threshold energy can be determined experimentally and the result is used to find the value of Q. If $Q > 0$ (exothermic reaction), there is no threshold condition and the reaction will occur even for very small energy.

Finally, if the reaction reaches the excited state of Y, the *Q-value* equation should include the mass energy of the excited state:

$$\begin{aligned} Q_{ex} &= (M_X + m_a - M_Y^* - m_b)c^2 \\ &= Q_0 - E_{ex} \end{aligned} \quad 3.11$$

where Q_0 is the *Q-value* corresponding to the ground state of Y, while Q_{ex} and E_{ex} are the excitation Q-value and energy above the ground state respectively, then the mass energy relation is:

$$M_Y^* c^2 = M_Y c^2 + E_{ex} \quad 3.12$$

3.3. Types of Radioactivity

A nucleus in an excited state is unstable because it can always undergo a transition (decay) to a lower-energy state of the *same or different nucleus*.

A nucleus which undergoes a transition spontaneously, that is, without being supplied with additional energy as in bombardment, is said to be radioactive. It is found experimentally that naturally occurring radioactive nuclides emit one or more of the three types of radiations, α- particles, β- particles, and γ-rays.

3.3.1. Alpha decay (α)

Alpha decay is the emission of alpha particles (helium nuclei) which may be represented as either ${}_2^4He$ *or* ${}_2^4\alpha$. When an unstable (parent) nucleus disintegrates to a daughter through ejection an alpha particle, the atomic number is reduced by 2 and the mass number decreased by 4, and the transition is:

$$ {}_Z^AX \rightarrow {}_{Z-2}^{A-4}Y + {}_2^4He \qquad 3.13 $$

If we assume that the parent nucleus is initially at rest, which is a common case, the conservation of energy requires:

$$ M_pc^2 = M_dc^2 + T_d + M_\alpha c^2 + T_\alpha \qquad 3.14 $$

where M_p, M_d *and* M_α are the masses of the parent, daughter and α particle respectively. Similarly, T_d *and* T_α represent the kinetic energies of the daughter and of the α-particle.

Eq.3.2 can be written as:

$$ T_d + T_\alpha = (M_p - M_d - M_\alpha)\, c^2 = \Delta Mc^2 = Q \qquad 3.15 $$

Here, we use atomic masses instead of nuclear masses since the masses of electrons cancel. We define the disintegration energy *Q-value* as the difference in the rest masses of the initial and final states and obviously equal the sum of the kinetic energies of the final state particles. Applying the law of conservation of momentum:

$$P_i = P_f \qquad 3.16$$

where P_i *and* P_f are the initial and final momentum of reactants.

Now, since the parent nucleus, in α-decay, decays from the rest, then the daughter nucleus and the α-particle must necessarily move in opposite directions to conserve momentum. For non-relativistic particles, the momentum and kinetic energies can be written as:

$$M_d\, \upsilon_d = M_\alpha\, \upsilon_\alpha \text{ or } \upsilon_d = M_\alpha\, \upsilon_\alpha / M_d \qquad 3.17$$

$$T_\alpha = \tfrac{1}{2} M_\alpha\, \upsilon^2_\alpha, \text{ and } T_d = \tfrac{1}{2} M_d\, \upsilon^2_d = (M_\alpha / M_d) T_\alpha \qquad 3.18$$

In almost all α-decay cases, the mass of the daughter nucleus is much greater than that of the α-particle. Then $\upsilon_d << \upsilon_\alpha$, and consequently the kinetic energy of the daughter nucleus is far smaller than that of the α particle. υ_d can be eliminated by Eq. 3.17, then the expressions for T_α in terms of the *Q-value*, using Eq. 3.15 and 3.18, is:

$$T_\alpha = \frac{M_d}{M_\alpha + M_d} Q = \frac{1}{1 + \frac{M_\alpha}{M_d}} Q \qquad 3.19$$

The kinetic energy of the emitted α particle cannot be negative, that is, $T_\alpha \geq 0$. Consequently, for α-decay to occur, we must have an exothermic process.

$$\Delta M \geq 0, \qquad Q \geq 0 \qquad 3.20$$

As α-decay mainly occurs in heavy nuclei, most of the energy is carried off by the α-particle. The kinetic energy of the daughter nucleus is then obtained from Eqs. 3.15 and 3.19.

$$T_d = Q - T_\alpha = \frac{M_\alpha}{M_\alpha + M_d} Q = \frac{M_\alpha}{M_d} T_\alpha \qquad 3.21$$

If we approximate the atomic masses of the nuclei by their mass numbers, the masses ratio can be approximated $M_\alpha/M_d \approx 4/(A-4)$. We can then rewrite Eq.3.21 as:

$$T_\alpha \approx \frac{A-4}{A} Q$$
$$T_d \approx \frac{4}{A} Q \qquad 3.22$$

It is clear from Eq.3.19 that the kinetic energy of the α-particle in the decay is unique. Careful measurements, however, have revealed a fine splitting in the energies of α-particles emitted from radioactive material, corresponding to possibly different *Q-values*. The most energetic α-particles are observed to be produced alone. Less energetic α-particles are always accompanied by the emission of gamma photons, i.e. the parent nucleus can transform to an excited state of the daughter nucleus, in which the *Q-value* will be lower by a value of the gamma energies. An example is uranium-234, which decays by the ejection of an alpha particle accompanied by the emission of a 0.068 MeV gamma.

$${}^{234}_{92}U \rightarrow {}^{230}_{90}Th + {}^{4}_{2}\alpha + \gamma + T$$

The combined kinetic energy of the daughter nucleus (Thorium-230) and the α-particle is designated as kinetic energy. The sum of the *KE* and the gamma energy is equal to the difference in mass between the original nucleus (Uranium-234) and the final particles (equivalent to the binding energy released, since $\Delta mc^2 = BE$ *or Q-value*). The alpha particle will carry off as much as 98% of the kinetic energy and, in most cases, can be considered to carry off all the kinetic energy.

3.3.2. Beta decay (β)

The theory of *beta decay* was developed by Fermi (1934) in analogy with the quantum theory of electromagnetic decay. Our concern is not the elements of this theory; rather we will be content to mention just one aspect of the theory, that concerning the statistical factor describing the momentum and energy distributions of the emitted β particles.

Beta decay is considered to be a weak interaction since the interaction potential is ~ 10^{-6} that of nuclear interactions, which are generally regarded as strong. β-decay is the emission of electrons of nuclear rather than orbital origin. These particles are electrons that have been expelled by excited nuclei and may have a charge of either sign e^-, e^+. β -decay is the most common type of radioactive decay, all nuclides not lying in the valley of stability are unstable against this transition.These electrons are emitted with a continuous spectrum of energies, as shown in Fig. 3.2.

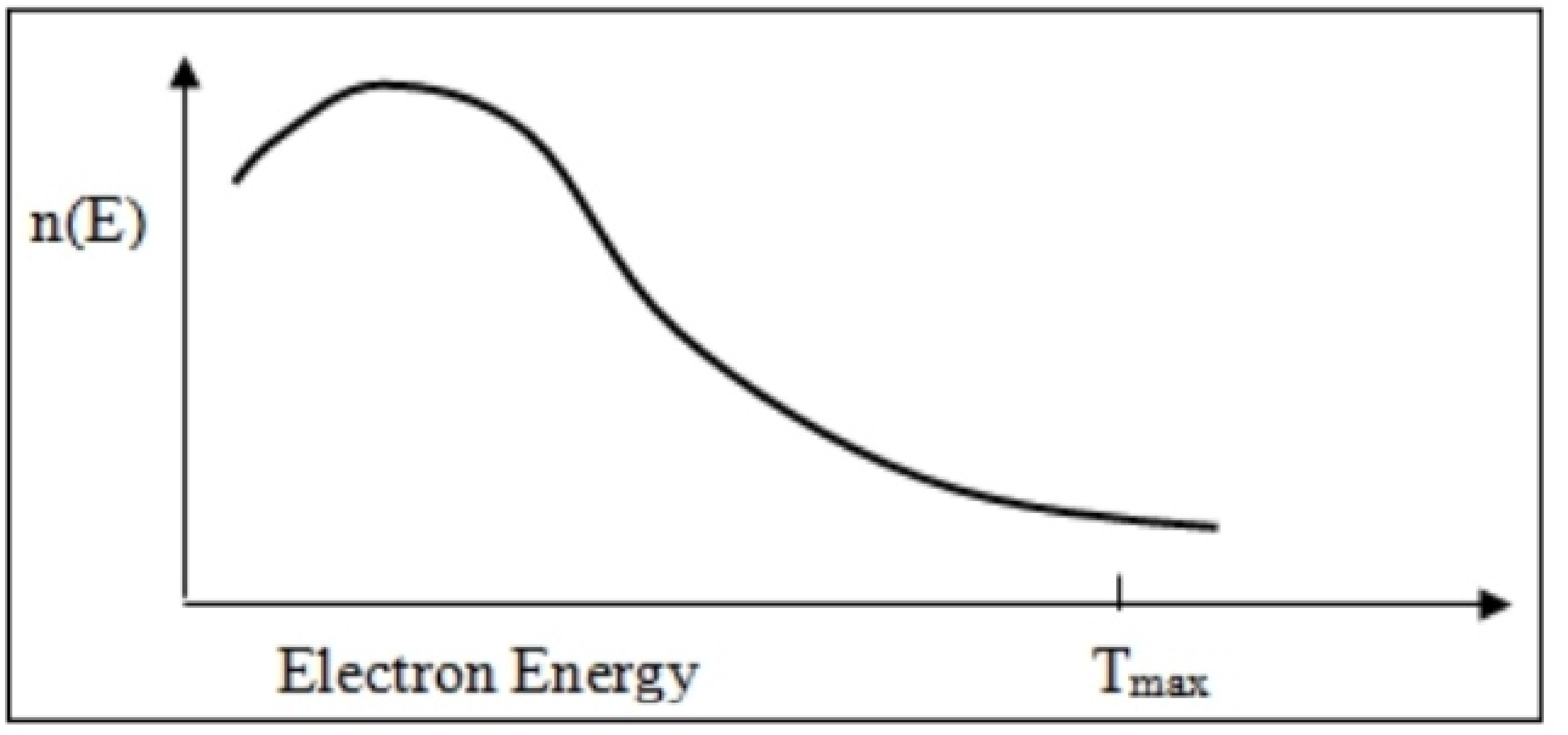

Figure 3.2. The observed differential distribution in the number of emitted electrons as a function of their energy.

If both energy and momentum are to be conserved, a third type of particle, the neutrino (υ) must be involved. The neutrino is associated with the positive electron emission, and its antiparticle, the antineutrino ($\bar{\upsilon}$) is emitted with a negative electron. These uncharged particles have only the weakest interaction with matter, no charge, neglecting its small mass (no mass as proposed by Pauli), and travel at the speed of light. For all practical purposes, they pass through all materials with so few interactions that the energy they possess cannot be recovered. The neutrinos and antineutrinos are included here only because they carry a portion of the kinetic energy that would otherwise belong to the beta particle, and therefore, must be considered for energy and momentum to be conserved. Also, linear and angular momenta are now conserved. They are normally ignored since they are not significant in the context of nuclear reactor applications.

3.3.2.1. β^- decay:

A nucleus with an over abundance of neutrons (with value of *N/Z* greater than that for stable nuclei, as shown in the chart of nuclides Fig. 2.2 and Fig. 3.5) can transform to a more stable nucleus as one excess neutron converts to a proton inside the nucleus and emit a negative electron associated with antineutrino. This kind of process is known as e^- or β^- decay and the transformation can be denoted by:

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \overline{\upsilon} \qquad 3.23 $$

If the parent nucleus decays from rest, the conservation of energy for electron emission will yield:

$$ M_p c^2 = T_d + M_d c^2 + T_e + m_e c^2 + T_{\overline{\upsilon}} + m_{\overline{\upsilon}} c^2 $$

or

$$ T_d + T_e + T_{\overline{\upsilon}} = \{M_p - M_d - m_e - m_{\overline{\upsilon}}\} c^2 $$

$$ = \Delta M c^2 = Q \qquad 3.24 $$

where M_p, M_d, m_e and $m_{\overline{\upsilon}}$, are respectively, the masses of the parent nucleus, the daughter nucleus, the electron and the antineutrino.

Similarly, T_d, T_e and $T_{\overline{\upsilon}}$ represent the kinetic energies of the decay daughter nucleus, the electron and antineutrino, respectively. We see from Eq. 3.24 that electron emission can take place only if the disintegration energy Q is positive, that is, when the mass of the parent nucleus is greater than the sum of the masses of the decay products. In fact, neglecting small differences in atomic binding energies, we conclude that electron emission will take place if:

$$Q = \{M(A, Z) - M(A, Z + l) - m_{\bar{\upsilon}}\}\, c^2$$

$$\approx \{M(A,Z)-M\{A,Z+1)\}\, c^2 \geq 0 \qquad 3.25$$

Eq. 3.25 represents the atomic masses, including the mass of atomic electrons, with neglecting the small mass of the neutrino. Furthermore, because the daughter nucleus is much heavier than either the electron or the antineutrino, the small recoil energy of the daughter can be ignored, and for any β^- decay we can write:

$$T_e + T_{\bar{\upsilon}} \approx Q \qquad 3.26$$

Eq.3.26 shows clearly that, with a $\bar{\upsilon}$ in the final state, the energy of the electron is no longer unique. In fact, any continuous value $0 \leq T_e \leq Q$ is kinematically allowed and the maximum electron energy, corresponding to $T_{\bar{\upsilon}} = 0$, is given by the endpoint value of Eq. 3.26.

$$(T_{e^-})_{max} = Q \qquad 3.27$$

Pauli's postulate therefore accommodates the continuous energy spectrum in β^- decay, and simultaneously restores all the accepted conservation laws.

3.3.2.2. β^+ decay:

Positively charged electrons (beta-plus) are known as positrons β^+ (electron antiparticle). Except for sign, both β^+ *and* β^- are nearly identical. The positron, represented as e^+, or β^+, is ejected from the nucleus (with value of *N/Z* less than that for stable nuclei, Fig. 2.2 and 3.5), the atomic number of the nucleus decreased by one and the mass number remains unchanged. A proton converts to a neutron by emission of β^+ and neutrino υ. The process represented as:

$$ {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + e^+ + \upsilon \qquad 3.28 $$

Similar to the β^- decay, the disintegration energy for positron emission is given by:

$$ Q = (M_p - M_d - m_{e+} - m_{\upsilon})\, c^2 $$

$$ = (M(A,Z) - M(A,Z\text{-}l) - 2m_e\text{-}m_{\upsilon})\, c^2 $$

$$ \approx (M(A,Z) - M(A,Z\text{-}l) - 2m_e)\, c^2 \qquad 3.29 $$

Where, again, all the *M (A, Z)* in the last line of Eq. 3.29 refer to full atomic weights, and *Q* must be positive for the decay to occur.

3.3.2.3. *Electron capture* (*K-capture*)

Nuclei having an excess of protons may capture an electron from one of the inner orbits, which immediately combines with a proton in the nucleus to form a neutron. This process is called *electron capture* (EC). The electron is normally captured from the innermost orbit (the K-shell), and, consequently, this process is sometimes called K-capture. This process can be represented as:

$$ {}_{Z}^{A}X + e^- \rightarrow {}_{Z-1}^{A}Y + \upsilon \qquad 3.30 $$

A neutrino is formed at the same time that the neutron is formed and energy carried off by it serves to conserve momentum. Any energy that is available due to the atomic mass of the product being appreciably less than that of the parent will appear as gamma radiation. In addition, there will always be characteristic x-rays given off when an electron from one of the higher energy shells moves in to fill the vacancy in the K-shell. Electron capture is shown graphically in Fig. 3.3.

Electron capture and positron emission result in the radioactive production of the same daughter product, and they exist as competing processes. For positron emission β^+ to occur, however, the mass of the daughter product must be less than the mass of the parent by an amount equal to at least twice the mass of an electron Eq.3.29. This mass difference between the parent and daughter is necessary to account for two items present in the parent but not in the daughter. One item is the positron ejected from the nucleus of the parent. The other item is that the daughter product has only one less orbital electron than the parent does. If this requirement is not met, orbital electron capture takes place exclusively.

Similarly, electron capture process can take place only if:

$$
\begin{aligned}
Q &= (M_p + m_e - M_d - m_\upsilon)\, c^2 \\
&= \{M(A, Z) - M(A, Z\text{-}1) - m_\upsilon\}\, c^2 \\
&\approx \{M(A,Z) - M(A,Z\text{-}1)\}\, c^2 \geq 0 \qquad 3.31
\end{aligned}
$$

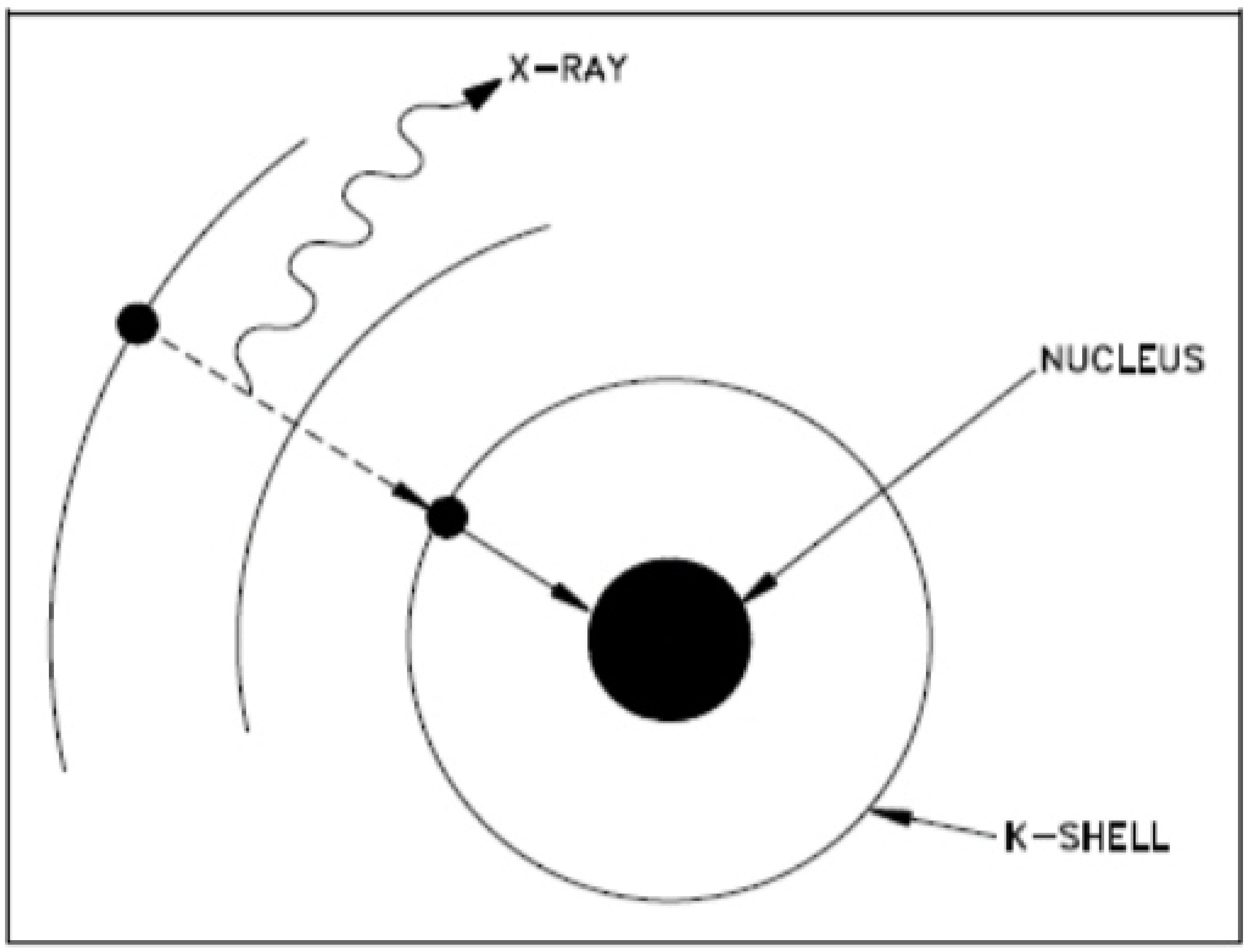

Figure 3.3. Orbital Electron Capture.

3.3.2.4. Selection rules for beta decay

For each nuclear level, there is an assignment of spin and parity. This information is essential for determining whether a transition of a particular transition between initial and final states is allowed according to certain selection rules of quantum mechanics. Also if a transition is allowed, what mode of decay is most likely?

Consider a beta transition between two nuclear states of well defined angular momentum with the emission of, say, a positron and neutrino or an electron and an antineutrino. In this transition, the total angular momentum and the parity of the angular momentum states must be conserved; see Ch. 2. Then the angular momentum and parity are generally expressed in beta decay P → D + β +υ as:

$$\boldsymbol{j}_P = \boldsymbol{j}_D + \boldsymbol{L}_\beta + \boldsymbol{S}_\beta\ ,\ L = 0, 1, 2 \ldots and\ S = 0\ and\ 1 \quad 3.32$$

$$\pi_P = \pi_D(-1)^{L_\beta} \quad 3.33$$