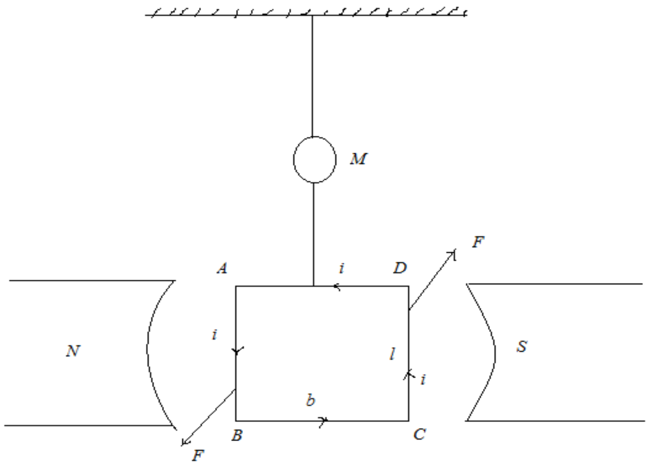
LECTURE-9 (Pabitra Halder, Assistant Professor, Department of Physics, Berhampore Girls' College)

Electromagnetic Measuring Instruments

Ballistic galvanometer:

A ballistic galvanometer is used to measure an amount of charge q which passes through the galvanometer coil for a very short interval of time (T_0).

Construction:



The ballistic galvanometer consists of a rectangular coil ABCD having n turns of wire wound on a non conducting frame. The coil is suspended by a suspension fibre in a uniform radial magnetic field of flux density $\vec{B}$ provided by a permanent magnet whose pole pieces are N and S. The suspension fibre has a small mirror M. The deflection of the coil can be measured by the usual lamp and scale arrangement.

If a current of *i* ampere flows through the coil at any time t in the direction as shown in figure above, the sides AB and CD will experience equal and opposite force while the sides BC and DA will not be acted upon by any force.

Hence, the coil will experience a couple due to which it will undergo a deflection. The amount of charge q flowing through the galvanometer coil is measured in terms of the first deflection θ of the coil.

Theory:

Let AB=CD=l and BC=DA=b

Therefore the area of the coil = b l= α (say).

If dq amount of charge flows for an infinitesimal time interval dt, the corresponding current $i = \frac{dq}{dt}$.

Therefore, q= $\int_0^T i\, dt$

Due to current *i* ampere, the magnitude of the force experience AB (or CD) is

F= $\left|n(i\,\vec{l} \times \vec{B})\right|$ = n Bil ·············· (1)

Thus, the impulse imparted to each of the side AB and CD in the time interval T_0 is

$\int_0^{T0}$ n Bil dt = nBlq (since, i T_0= q)

Hence, the moment of the impulse = nBlq b = nBq α = angular momentum gained by the coil.

If I be the moment of inertia of the coil about the axis of rotation (*i.e* about the suspension fibre) and ω be the angular velocity attain by the coil, we may write

$$I\omega = nBq\,\alpha \cdots\cdots\cdots\cdots (2)$$

Since, the coil has been deflected by an angle θ, the work done against this restoring couple of the suspension fibre is

$\Gamma = \int_0^\theta C\,\theta\, d\theta$; where C is the couple per unit twist of the suspension fibre.

From conservation of energy

$\frac{1}{2} I\,\omega^2 = \frac{1}{2} C\,\theta^2 \rightarrow I\,\omega^2 = C\,\theta^2 \cdots\cdots\cdots\cdots (3)$

Due to restoring couple developed in the suspension fibre, the coil will execute torsional oscillation whose time period is given by

$T = 2\pi\sqrt{\frac{I}{C}} \rightarrow I = \frac{CT^2}{4\pi^2} \cdots\cdots\cdots\cdots (4)$

From equations (3) and (4), we have

$I^2\omega^2 = C\,\theta^2\ I = \frac{C^2T^2\theta^2}{4\pi^2}$

Using equation (2), we get

$n^2\,B^2\,\alpha^2 q^2 = \frac{C^2T^2\theta^2}{4\pi^2} \rightarrow q = \frac{CT}{2\pi n\alpha B}\,\theta \rightarrow q = K\,\theta \cdots\cdots\cdots\cdots (5)$

Where K is the galvanometer constant of magnitude ($\frac{CT}{2\pi n\alpha B}$). In this way q is estimated in terms of the deflection θ of the galvanometer coil.

Correction due to damping:

Though the electromagnetic damping in a ballistic galvanometer is made small by winding the wire on a non-conducting frame, some amount of damping is present mainly due to the resistance of air against the motion of the coil. If $\theta_1, \theta_2, \theta_3, \theta_4, \cdots$ be the successive throws of the coil then from the experiments,

We have, $\frac{\theta 1}{\theta 2} = \frac{\theta 2}{\theta 3} = \frac{\theta 3}{\theta 4} = \cdots\cdots = d$ (say)(where d is the decrement).

Let us put $d = e^{\lambda}$. So that $\boldsymbol{\lambda} = \ln d$. Thus $\boldsymbol{\lambda}$ is termed as the log decrement.

Now, $\frac{\theta 1}{\theta 3} = \frac{\theta 1}{\theta 2}\cdot\frac{\theta 2}{\theta 3} = e^{\lambda}.\, e^{\lambda} = e^{2\lambda}$

$\frac{\theta 1}{\theta 5} = \frac{\theta 1}{\theta 3}\cdot\frac{\theta 3}{\theta 4}\cdot\frac{\theta 4}{\theta 5} = e^{2\lambda}.\, e^{\lambda}.\, e^{\lambda} = e^{4\lambda}$

The above result implies that if θ_0 be the first throw of the galvanometer coil (which is observed after a quarter of a period) and θ_1 is observed value, we must have

$\frac{\theta_0}{\theta_1} = e^{\lambda/2} \rightarrow \theta_0 = \theta_1\, e^{\lambda/2} = \theta_1\,(1 + \frac{\lambda}{2} + \frac{\lambda^2}{4.2.1} + \cdots\cdots)$

$\theta_0 \approx = \theta_1\,(1+\frac{\lambda}{2}) \cdots\cdots$ (6) , since $\boldsymbol{\lambda}$ is very small.

Thus, if θ be the observed first throw of the galvanometer coil the corrected expression for the charge traversing the coil is given by

$q = K\,\theta\,(1+\frac{\lambda}{2}) \cdots\cdots$ (7)

$q = \frac{CT}{2\pi n\alpha B}\,\theta_1(1+\frac{\lambda}{2}) \cdots\cdots$ (8), where θ_1 represents the first throw of the galvanometer.

Current, charge and voltage sensitivities of a moving coil galvanometer:

Current sensitivity (Figure of merit):

The figure of merit or current sensitivity of a moving coil galvanometer is the current required to produce a deflection of 1 mm on a scale kept at a distance of 1 m from the mirror. It is expressed in μA/mm.

Current sensitivity = $\frac{i}{\theta} = \frac{C}{n\alpha B}$.

Charge sensitivity:

The charge sensitivity (the ballistic reduction factor) of a moving coil galvanometer is the charge (transient current) required to produce a deflection (throw or kick) of 1 mm on a scale kept at a distance of 1 m from the mirror.

Charge sensitivity, K = $\frac{q}{\theta} = \frac{CT}{2\pi n\alpha B} = \frac{T}{2\pi} \times \frac{C}{n\alpha B} = \frac{T}{2\pi} \times$ Current sensitivity

Some authors defined the current sensitivity as the deflection produced by unit current on a scale kept at a distance 1m from the mirror of the galvanometer.

Current sensitivity = $\frac{\theta}{i}$

With this definition the figure of merit is the reciprocal of the current sensitivity.

Voltage sensitivity:

The voltage sensitivity is the potential difference that should be applied to the galvanometer to produce a deflection of 1 mm on a scale placed at a distance of 1 m from the mirror. It is expressed in μV/mm.

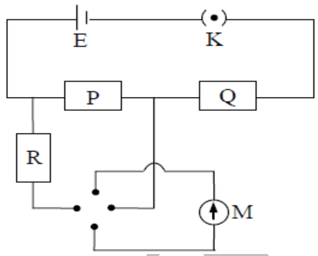


Figure shows the electrical circuit for the determination of current and voltage sensitivities of a moving coil galvanometer. Resistance boxes P and Q connected in series with the lead accumulator E form a potential divider arrangement. Potential difference developed across P is applied to the moving coil galvanometer through the resistance R and the commutator. A low resistance, say 1 Ω, is introduced in P and a high resistance, say 9999 Ω, in Q. The deflection produced θ is determined. Then,

Voltage sensitivity, $S_v = \frac{EP}{(P+Q)\theta} \times 10^6$ μV/mm.

Now resistance in R is adjusted such that the deflection becomes θ/2. The resistance R is, then, equal to the galvanometer resistance R_g.

Current sensitivity, $S_c = \frac{EP}{(P+Q)R_g\theta} \times 10^6$ μA/mm.

The experiment is repeated for different values of P keeping P+Q equal to 10000 Ω.

(Remember, θ is originally defined as the angle of deflection of the mirror. When the mirror turns through an angle θ, the reflected ray turns through an angle 2θ. Since the distance between the mirror and the scale is 1 m, angle in radian = arc length in metre. Thus, if θ is taken as the scale reading in millimetre, multiplication with 2 is needed in the calculations of charge sensitivity, current sensitivity and voltage sensitivity).

Let d be the deflection in mm on the scale, then $\Phi = 2\theta = \frac{d}{D} \times 10^{-3}$ radian = d $\times 10^{-3}$ radian. (Since, D=1 m)

Then, $\frac{I}{\Phi} = \frac{I \times 10^3}{d}$ ampere per metre = $\frac{I}{d}$ ampere per millimetre.

<u>Electromagnetic damping in a moving coil galvanometer:</u>

A moving coil galvanometer consists of a rectangular current carrying coil suspended in a uniform radial magnetic field. The current through the coil produces a torque which tries to rotate the coil. A restoring torque, provided by the stiffness of the suspension, is set up in the system and it counterbalances the electromagnetic torque.

<u>Damping factors:</u>

1. Viscous drag of air and mechanical friction. The damping couple due to this is proportional to the angular velocity of the system. Usually it is very small and is neglected.

2. Induced currents in the neighbouring conductors. These currents produce two types of damping couple, the open circuit damping couple and the closed circuit damping couple. The former one, according to the law of electromagnetic induction, is proportional to the angular velocity and is represented by $-b\frac{d\theta}{dt}$, where, b is the damping coefficient. The latter one is directly proportional to the angular velocity and inversely proportional to the resistance of the circuit. It is given by, $-\frac{\xi}{R}\frac{d\theta}{dt}$, where, ξ involves all the coil constants such as area, magnetic flux and so on.

If C is the restoring couple per unit twist of the suspension and I is the moment of inertia of the oscillating system, the equation of motion is given by,

$$I\frac{d^2\theta}{dt^2} = -C\theta - b\frac{d\theta}{dt} - \frac{\xi}{R}\frac{d\theta}{dt} \rightarrow \frac{d^2\theta}{dt^2} + \frac{1}{I}\left(b+\frac{\xi}{R}\right)\frac{d\theta}{dt} + \frac{C}{I}\theta = 0$$

Now put, $\frac{1}{I}\left(b+\frac{\xi}{R}\right) = \gamma_e$ and $\frac{C}{I} = \omega_0^2$

Then, $\frac{d^2\theta}{dt^2} + \gamma_e\frac{d\theta}{dt} + \omega_0^2\,\theta = 0$

This equation is similar to the equation for a damped harmonic oscillator given by,

$\ddot{x} + 2r\,\dot{x} + \omega_0^2\,x = 0$

Comparing these equations we get, $x = \theta$, $2r = \gamma_e$ and $\omega_0^2 = \frac{C}{I}$

For damped oscillations, we have, $x = e^{-rt}\left\{C_1 e^{\left(\sqrt{r^2-\omega_0^2}\right)t} + C_2 e^{\left(-\sqrt{r^2-\omega_0^2}\right)t}\right\}$

Thus, $\theta = e^{-\frac{\gamma_e}{2}t}\left\{C_1 e^{\left(\sqrt{\frac{\gamma_e{}^2}{4}-\omega_0^2}\right)t} + C_2 e^{\left(-\sqrt{\frac{\gamma_e{}^2}{4}-\omega_0^2}\right)t}\right\}$ .... (1)

Where, C1 and C2 are undetermined constants to be evaluated from the initial conditions.

Case 1: Non-oscillatory, aperiodic or dead beat motion:

If the damping is high such that $\frac{{\gamma_e}^2}{4} > \omega_0^2$ the exponents of equation (1) are real and the motion of the system is non-oscillatory. The angular displacement decays according to equation (1). This is the case of dead beat motion. The requirements of a dead beat galvanometer are,

1. The moment of inertia I of the system must be small.
2. The electromagnetic rotational resistance 'b' should be large and the suspension is not fine.
3. The term ξ, which involves all coil constants, should be large and the coil should be wound on a conducting frame.
4. The resistance R must be small.

Case 2: Critical damping:

When $\frac{{\gamma_e}^2}{4} = \omega_0^2$, the galvanometer is said to be critically damped. The motion of the coil is non-oscillatory and it comes to rest in a minimum time after deflection.

Case 3: Light damping: Ballistic motion:

When $\frac{{\gamma_e}^2}{4} < \omega_0^2$, the exponents of the bracketed terms of eqn.20 are imaginary. Hence the motion of the coil is oscillatory in this case. The solution is given by

$$\theta = e^{-\frac{\gamma_e}{2}t} \left\{ C_1 e^{\left(i\sqrt{\omega_0^2 - \frac{{\gamma_e}^2}{4}}\right)t} + C_2 e^{\left(-i\sqrt{\omega_0^2 - \frac{{\gamma_e}^2}{4}}\right)t} \right\}$$

$$\theta = Q_0\, e^{-\frac{\gamma_e}{2}t} \sin\left\{\left(\sqrt{\omega_0^2 - \frac{{\gamma_e}^2}{4}}\right)t + \Phi_0\right\} \rightarrow \theta = Q_0\, e^{-\frac{\gamma_e}{2}t} \sin(qt + \Phi_0)$$

Where, $q = \sqrt{\omega_0^2 - \frac{{\gamma_e}^2}{4}} = \sqrt{\frac{C}{I} - \frac{1}{4I^2}\left(b + \frac{\xi}{R}\right)^2}$

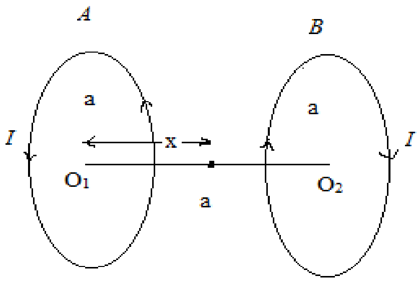
The motion of the galvanometer is said to be ballistic when the damping factor $\frac{\gamma_e}{2} = \frac{1}{2I}\left(b + \frac{\xi}{R}\right)$ is small. The requirements of a ballistic galvanometer are reverse of those for a dead beat galvanometer and are,

1. The moment of inertia I of the system must be large.
2. The electromagnetic rotational resistance 'b' is small and the suspension is fine.
3. The term ξ, which involves all coil constants, should be small and the coil should be wound on a non-conducting frame like wood or paper.
4. The resistance R must be large.

Uses of ballistic galvanometers:

1. To compare capacities of capacitors.
2. To compare e m f of cells.
3. To find self and mutual inductance of coils.
4. To find the magnetic flux or to find the intensity of a magnetic field.
5. To find the angle of dip at a place using earth inductor.
6. To find a high resistance, by method of leakage through a capacitor.

Helmholtz double coil galvanometer:



Construction:

In the Helmholtz double coil galvanometer two identical circular coil of radius a each having n turns of wire wound over it are placed co-axially at a distance of separation equal to the radius of each coil. The current through each coil flowing in such a direction the magnetic inductions at any intermediate point between the coils due to both the coils are additive. If small magnetic needle is placed join at the midpoint of the line O_1O_2 joining the centre of two coils.

Theory:

We know that the magnetic induction at any axial point due to a single circular coil carrying a current of *I* ampere is given by

$|\vec{B}|$ = B = $\frac{\mu_0 n I a^2}{2(a^2+x^2)^{3/2}}$ wb/m^2 .... (1)

Where x is the distance of the axial point from the centre of the coil.

We now find a point on the axis where the rate of change of B with respect to x becomes uniform, so that the total induction due to both the coil in a small region about that point remains constant.

Hence, for that point $\frac{dB}{dx}$ remains constant for a single coil.

Therefore, $\frac{d^2B}{dx^2}$ = 0

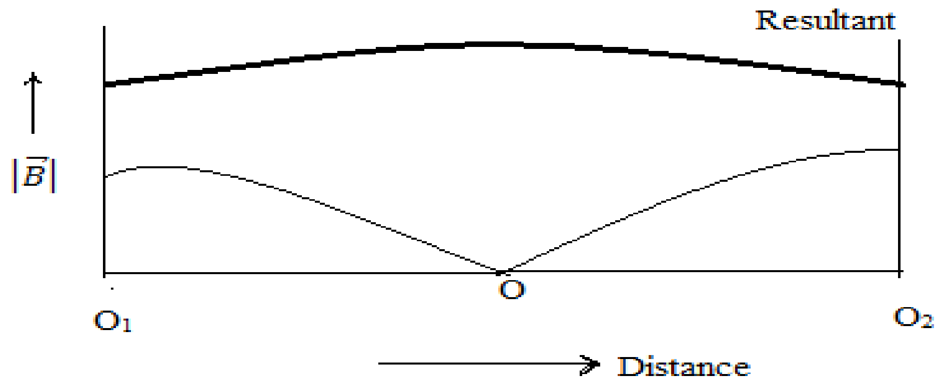
From (1), B = K $(a^2+x^2)^{-3/2}$, where K= $\frac{\mu_0 n I a^2}{2}$ = constant.

$\frac{dB}{dx}$ = - $\frac{3K}{2}$ $(a^2+x^2)^{-5/2}$ × 2x = - 3K $(a^2+x^2)^{-5/2}$ x

$\frac{d^2B}{dx^2}$ = $\frac{15K}{2}$ $(a^2+x^2)^{-7/2} \times 2x^2$ - 3K $(a^2+x^2)^{-5/2}$ = 3K $(a^2+x^2)^{-7/2}$ $[5x^2-a^2-x^2]$

Thus, if $\frac{d^2B}{dx^2}$ = 0; $4x^2-a^2$ = 0 → x = $\pm\frac{a}{2}$ .... (2)

Hence, the rate of change of magnetic induction becomes uniform at a point midway between the coil A and B. Thus, when two coils are those the rate of decrement of the induction due to one coil balance by the rate of increment of the induction due to the other coil, as one moves away from the midpoint. The graphical variation of $|\vec{B}|$ with the distance from the centre of a coil and also the resultant induction due to the two coils is shown below



As evident from above graph, the resultant induction remains uniform over a finite region about the midpoint of the two coils. The total magnetic induction at a midpoint due to both the coil is

$$\left|\overrightarrow{B_1}\right| = \frac{2\mu_0 n I a^2}{2(a^2+\frac{a^2}{4})^{3/2}} \text{ wb/m}^2 = \frac{8\mu_0 n I}{5\sqrt{5}\ a} \text{ wb/m}^2$$

If a magnetic needle is placed at the midpoint of the coil and if θ be the equilibrium deflection of the needle we have

$\frac{8\mu_0 n I}{5\sqrt{5}\ a} = \text{H}\ \mu_0 \tan\theta$; where H is the horizontal component of the intensity of earth's magnetic field.

Therefore, $I = \frac{5\sqrt{5}\ a\ \text{H}\ \tan\theta}{8\ n} = K' \tan\theta$; Where $K' = \frac{5\sqrt{5}\ a\ \text{H}}{8\ n}$

Advantage of a Helmholtz double coil galvanometer:

1. In an ordinary tangent galvanometer the field due to the coil is assumed to remain over a finite length. But it is not true. So in an ordinary tangent galvanometer we should use a small magnetic needle. But in Helmholtz double coil galvanometer this difficulty is removed. Since, the field remains uniform over a finite length.

2. In ordinary tangent galvanometer, the deflection of the needle is small for a weak current. But it is relatively large in a double coil galvanometer.

3. Helmholtz double coil galvanometer finds an important application in the absolute determination of current and also for the accurate determination of earth's horizontal and vertical component of intensity.