```python
"""
Program to show orthonormality relation of Legendre Polynomials.
Trapezoidal & Simpson Rule is used to integrate
"""
from scipy.special import legendre
from scipy.integrate import quad

def integrand(x,m,n):
    Pm = legendre(m)
    Pn = legendre(n)
    Fm = Pm(x)
    Fn = Pn(x)
    return Fm*Fn

def trapezoid(a,b,m1,n1):
    n = 2000
    h = (b-a)/(n-1)
    s = 0.5*(integrand(a,m1,n1) + integrand(b,m1,n1))
    for i in range(1,n-1):
        s += integrand(a+i*h,m1,n1)
    return   h*s

def simpson(a,b,m1,n1):
    n = 2000
# increment n if even
    if (n % 2 == 0):
        n += 1
    h = (b-a)/(n-1)
    s1 = s2 = 0
    for i in range(2,n-2,2):
        s1 += integrand(a + i*h,m1,n1) # odd-index sum
    for i in range(1,n-1,2):
        s2 += integrand(a + i*h,m1,n1) # even-index sum
    return (h/3)*(integrand(a,m1,n1) + 4*s2 + 2*s1 + integrand(b,m1,n1))

m = 2
n = 2

I_trapz = trapezoid(-1,1,m,n)
I_simp = simpson(-1,1,m,n)

# From orthonormality condition when n != m --> Integral = 0
# When n == m --> Integral = 2/(2n+1)
if m == n:
    exactValue = 2.0/(2.0*n+1)
else:
    exactValue = 0

print('m =', m, 'n = ', n)
print('Integral of Pm*Pn using Trapezoidal Rule = ', I_trapz)
print('Integral of Pm*Pn using Simpson Rule = ', I_simp)
print('Orthonormal value of integral of Pm*Pn = ', exactValue)
```

```
m = 2 n =  2
Integral of Pm*Pn using Trapezoidal Rule =  0.40000100100059977
Integral of Pm*Pn using Simpson Rule =  0.40000000000059965
Orthonormal value of integral of Pm*Pn =  0.4
```