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"""
Program to show orthonormality relation of Legendre Polynomials.
Trapezoidal & Simpson Rule is used to integrate
 Evaluates the n-th order Legendre polynomial of x using the recurrence
 relation:
 (2n+1)*x*P_n(x) = (n+1)*P_n+1(x) + n*P_n-1(x), n = 1,2,3,...
 We start the generation of successive Legendre Polynomial terms by first
 specifying P_0(x) = 1 and P_1(x) = x and start the iteration from third term
 onward  so we start by replacing n by n-1 in the above recurrence relation:
(2n-1)*x*P_n-1(x) = n*P_n(x) + (n-1)*P_n-2(x)
=> P_n(x) = ((2n-1)*x*P_n-1(x) - (n-1)*P_n-2(x))/n
P_n(x) ---> f
P_n-1(x) = f1
P_n-2 (x) = f2

------------------------------------------------------------------------------
"""
def Legendre(n, x):
    if n == 0:
        f = 1
    else:
        f = x
        f1 = 1
    for i in range(2,n+1):
        f2 = f1
        f1 = f
        f = ((2*i-1)*x*f1 - (i-1)*f2)/i
    return f

def integrand(x,m,n):
    Pm = Legendre(m,x)
    Pn = Legendre(n,x)
    Fm = Pm
    Fn = Pn
    return Fm*Fn

def trapezoid(a,b,m1,n1):
    n = 200
    h = (b-a)/(n-1)
    s = 0.5*(integrand(a,m1,n1) + integrand(b,m1,n1))
    for i in range(1,n-1):
        s += integrand(a+i*h,m1,n1)
    return  h*s

def simpson(a,b,m1,n1):
    n = 200
# increment n if even
    if (n % 2 == 0):
        n += 1
    h = (b-a)/(n-1)
    s1 = s2 = 0
    for i in range(2,n-2,2):
        s1 += integrand(a + i*h,m1,n1) # odd-index sum
    for i in range(1,n-1,2):
        s2 += integrand(a + i*h,m1,n1) # even-index sum
    return (h/3)*(integrand(a,m1,n1) + 4*s2 + 2*s1 + integrand(b,m1,n1))

m = 2
n = 2

I_trapz = trapezoid(-1,1,m,n)
I_simp = simpson(-1,1,m,n)

# From orthonormality condition when n != m --> Integral = 0
# When n == m --> Integral = 2/(2n+1)
if m == n:
    exactValue = 2.0/(2.0*n+1)
else:
    exactValue = 0

print('m =', m, 'n = ', n)
print('Integral of Pm*Pn using Trapezoidal Rule = ', I_trapz)
print('Integral of Pm*Pn using Simpson Rule = ', I_simp)
print('Orthonormal value of integral of Pm*Pn = ', exactValue)
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m = 2 n =  2
Integral of Pm*Pn using Trapezoidal Rule =  0.400101006019935 65
Integral of Pm*Pn using Simpson Rule =  0.400000005999999 96
Orthonormal value of integral of Pm*Pn =  0.4
```