Algorithm to compute power series representation of sine(x)

The sine series is computed using recurrence relation:

$$\frac{(N+1)^{th}Term}{(N)^{th}Term} = \frac{x^{2n+1}}{x^{2n-1}}\frac{(2n-1)!}{(2n+1)!} = \frac{x^2}{2n(2n+1)}$$

Remarks: Magnitude of x to be made less than π for faster series convergence. Convergence is reached when the sum of the series is not changing more than the allowed error (ϵ), as new term is added. The allowed error (ϵ) is read in as data and it represents the relative error = (term)/(series sum).

1. Read x, π, ϵ

Remarks: x is the argument, ϵ is allowed error.

2. if $|x| > \pi$ then

    if $x > 0$ then

        $$x \longleftarrow \left[\frac{x}{2\pi}\right]_{remainder}$$

    else

        $$x \longleftarrow -\left[\frac{x}{2\pi}\right]_{remainder}$$

    endif

  endif

3. $sum \longleftarrow x$
4. $term \longleftarrow x$
5. $factor \longleftarrow -x^2$
6. $n \longleftarrow 1$
7. repeat until $\left|\frac{term}{sum}\right| \leq \epsilon$

    $$denom \longleftarrow 2n \times (2n+1)$$

    $$term \longleftarrow \frac{term \times factor}{denom}$$

    $$sum \longleftarrow sum + term$$

    $$n \longleftarrow n + 1$$

8. Write $'sin(x) =', sum$

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"""
Program to compute sin(x)
  x --> Argument of sine function
  Magnitude of x to be made less than pi for faster convergence of series
  sin(x - 2*n*pi) = sin(x)
  Recurrence relation ==> (N=1)th_term/Nth_term = x**2/((2*n+1)*(2n))
  Convergence criterion --> sum of the series not changing adding a new term
  epsilon --> allowed error
  Relative error --> term/Series_Sum
"""
import numpy as np

x = 6
epsilon = 0.5e-06

if abs(x) > np.pi :
    if x > 0 :
        x = x%(2*np.pi)
    else:
        x = -x%(2*np.pi)
sum = x
term = x
factor = -x*x
n = 1

while abs(term/sum) >= epsilon :
    denom = (2*n)*(2*n+1)
    term = term*factor/denom
    sum = sum + term
    n = n + 1
print('Argument x of sine function in radian =',x)
print('sin(x) = ', sum, '   No. of terms =', n)
print('Exact value of sin(x) =', np.sin(x))
```

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Argument x of sine function in radian = 6.0
sin(x) =  -0.2794155022101806 3    No. of terms = 14
Exact value of sin(x) = -0.279415498198925 86
```