Suppes Chapter-9

SETS

Set / class / collection / aggregate

By A set we mean any kind of a collection of entities of any sort. The set of Indians or the set of integers.

The symbol $\in$ is used as 'belongs to'

1. Elizabeth II belongs to the class of woman

or simply

Elizabeth II $\in$ the class of women

In ordinary language

Elizabeth II is a woman

2. If A and B are sets which have exactly the same members, then $A = B$

$$A = B \leftrightarrow (x)\ (x \in A \leftrightarrow x \in B)$$

3. It is convenient to make our usage of the term 'set' wide enough to include empty sets, i.e., sets which have no members.

If two sets A and B are empty, then $A = B$

Λ = Empty set

Λ is the set such that for every x; x does not belong to Λ.

Symbolically :

$$(x) - (x \in A)$$

i.e.

$$(x)\ (x \notin A)$$

$\notin$ is used to indicate that something does not belong to a set.

We shall describe a set by writing down ~~the~~ names of its members, separated by commas, and enclosing the whole in braces (bracket).

1. $\{$Bhagirathi, Jalangi$\}$

2. $\{1, 3, 5\}$.

3. $\{1, 3, 5\} = \{1, 5, 3\}$

(Order is not important)

4. $\{1, 1, 3, 5\} = \{1, 3, 5\}$

(We do not count an element of a set twice)

5. $\{ \{1,2\}, \{3,4\}, \{5,6\} \}$

The set has just three members namely $\{1,2\}$, $\{3,4\}$ and $\{5,6\}$

The relation of Identity:

$$A = B$$

The relation of Identity is symmetric.

i.e. If $A = B$ then $B = A$

The relation of membership differs from the relation of Identity.

Because the relation of membership is not symmetric.

from $A \in B$ it does not follow that $B \in A$

For instance,

$$2 \in \{1,2\}$$

but $\{1,2\} \notin 2$

Moreover, The relation of Identity is transitive

i.e. If $A = B$ and $B = C$ then $A = C$

But the relation of membership is not transitive.

from $A \in B$ and $B \in C$ it does not follow that $A \in C$.

For example,

$$2 \in \{1, 2\}$$

and $\{1, 2\} \in \{\{1, 2\}, \{3, 4\}\}$

but $2 \notin \{\{1, 2\}, \{3, 4\}\}$

Identity $(=)$ is both symmetric and transitive.

But Membership $(\in)$ is ~~no~~ neither symmetric nor transitive.

Inclusion

If A and B are sets such that every member of A is also a member of B, then ~~A~~

A is a subset of B

i.e. A is included in B.

symbolically, $A \subseteq B$

for instance: The set of Indians is included in the set of men.

The set of ~~A~~ Indians $\subseteq$ the set of men.

symbolically,

$$A \subseteq B \Leftrightarrow (x)\ (x \in A \rightarrow x \in B)$$

The relation of inclusion is transitive;

i.e.

If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

The relation of Inclusion is not symmetric; ~~i.e.~~

$\{1, 2\} \subseteq \{1, 2, 3\}$.

but it is not the case that

$\{1, 2, 3\} \subseteq \{1, 2\}$

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1. Membership ($\in$) is neither ~~&~~ symmetric nor transitive.

2. Identity ($=$) is both symmetric and transitive.

3. Inclusion ($\subseteq$) is transitive but not symmetric.

Identity (=)

Symmetric : If $A = B$ then $B = C$

transitive : If $A = B$ and $B = C$

then $A = C$.

Membership ($\in$)

Not symmetric

From ~~If~~ $A \in B$ ~~an~~

it does not follow that

$B \in A$

Not transitive

From $A \in B$ and $B \in C$

it does not follow that

$A \in C$

Inclusion ($\subseteq$)

Not symmetric

If $A \subseteq B$

then it ~~no~~ is not the case that $B \subseteq A$

Transitive :

If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

<u>Proper subset ($\subset$) :</u>

when $A \subseteq B$, ~~and A = B ...~~ it may happends that $B \subseteq A$, so that A and B have exactly the same members. i.e. $A = B$

for example :— $\{1, 2, 3\} \subseteq \{1, 2, 3\}$

But when $A \subseteq B$ but $A \neq B$

then A is a proper subset of B.

symbolically, $A \subset B$

for example $\{1, 2\} \subset \{1, 2, 3\}$

<u>Subset</u> ($\subseteq$)

when $A \subseteq B$ ~~but~~ and $A = B$

~~But~~ <u>Proper subset</u> ($\subset$)

when $A \subseteq B$ but $A \neq B$