

## SETS

Set / class / collection / aggregate

By a set we mean any kind of a collection of entities of any sort. The set of Indians or the set of integers.

The symbol  $\in$  is used as 'belongs to'

1. Elizabeth II belongs to the class of women  
or simply

Elizabeth II  $\in$  the class of women

In ordinary language

Elizabeth II is a woman

2. If A and B are sets which have exactly the same members, then  $A = B$

$$A = B \leftrightarrow (x) (x \in A \leftrightarrow x \in B)$$

3. It is convenient to make our usage of the term 'set' wide enough to include empty sets, i.e., sets which have no members.

If two sets A and B are empty, then  
 $A = B$

$\Lambda$  = Empty set

$\Lambda$  is the set such that for every  $x$ ,  
 $x$  does not belong to  $\Lambda$ .

Symbolically :

$$(\neg x) \rightarrow (x \notin A)$$

i.e.

$$(\neg x) \rightarrow (x \notin A)$$

$\notin$  is used to indicate <sup>that</sup> something does not belong to a set.

We shall describe a set by writing down the names of its members, separated by commas, and enclosing the whole in braces - (bracket).

1.  $\{ \text{Bhagirathi, Jalangi} \}$

2.  $\{ 1, 3, 5 \}$

3.  $\{ 1, 3, 5 \} = \{ 1, 5, 3 \}$

(Order is not important)

4.  $\{ 1, 1, 3, 5 \} = \{ 1, 3, 5 \}$

(We do not count an element of a set twice)

$$5. \{ \{1, 2\}, \{3, 4\}, \{5, 6\} \}$$

The set has just three members namely  
 $\{1, 2\}$ ,  $\{3, 4\}$  and  $\{5, 6\}$

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The relation of Identity :

$$A = B$$

The relation of Identity is symmetric.

i.e. If  $A = B$  then  $B = A$

The relation of membership differs from  
the relation of Identity.

Because the relation of membership is not symmetric.

From  $A \in B$  it does not follow that  $B \in A$

For instance,

$$2 \in \{1, 2\}$$

but  $\{1, 2\} \notin 2$

So if  $A \in B$  then, it is not that  $B \in A$

Moreover, The relation of Identity is transitive

i.e. If  $A = B$  and  $B = C$  then  $A = C$

But the relation of membership is not transitive.

from  $A \in B$  and  $B \in C$  it does not follow that  $A \in C$ .

For example,

$$2 \in \{1, 2\}$$

$$\text{and } \{1, 2\} \in \{\{1, 2\}, \{3, 4\}\}$$

$$\text{but } 2 \notin \{\{1, 2\}, \{3, 4\}\}$$

Identity ( $=$ ) is both symmetric and transitive.

But Membership ( $\in$ ) is ~~ne~~ neither symmetric nor transitive.

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### Inclusion

If  $A$  and  $B$  are sets such that every member of  $A$  is also a member of  $B$ , then  $A$

$A$  is a subset of  $B$

i.e.  $A$  is included in  $B$ .

symbolically,  $A \subseteq B$

for instance: The set of Indians is included in the set of men.

The set of ~~As~~ Indians  $\subseteq$  the set of men.  
symbolically,

$$A \subseteq B \leftrightarrow (\forall x) (x \in A \rightarrow x \in B)$$

The relation of inclusion is transitive;  
i.e.

If  $A \subseteq B$  and  $B \subseteq C$  then  $A \subseteq C$ .

The relation of inclusion is not symmetric; ~~no~~.

$$\{1, 2\} \subseteq \{1, 2, 3\}$$

but it is not the case that

$$\{1, 2, 3\} \subseteq \{1, 2\}$$

1. Membership ( $\in$ ) is neither symmetric nor transitive.
2. Identity ( $=$ ) is both symmetric and transitive.
3. Inclusion ( $\subseteq$ ) is transitive but not symmetric.

Identity (=)

Symmetric: If  $A = B$  then  $B = A$

Transitive: If  $A = B$  and  $B = C$

then  $A = C$

Membership ( $\in$ )

Not symmetric

From ~~if~~  $A \in B$  ~~and~~

it does not follow that

$B \in A$

Not transitive

From  $A \in B$  and  $B \in C$

it does not follow that

$A \in C$

Inclusion ( $\subseteq$ )

Not symmetric

If  $A \subseteq B$

then it is not the case that  $B \subseteq A$

transitive :

If  $A \subseteq B$  and  $B \subseteq C$  then  $A \subseteq C$ .

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Proper subset ( $\subset$ ) :

When  $A \subseteq B$ , ~~and  $A = B$  also~~ it may happen, <sup>that  $B \subseteq A$ , so that</sup>  $A$  and  $B$  have exactly the

same members - i.e.  $A = B$

for example: —  $\{1, 2, 3\} \subseteq \{1, 2, 3\}$

But when  $A \subseteq B$  but  $A \neq B$

then  $A$  is a proper subset of  $B$ .

symbolically,  $A \subset B$

for example  $\{1, 2\} \subset \{1, 2, 3\}$

Subset ( $\subseteq$ )

when  $A \subseteq B$  ~~but~~ and  $A = B$

~~But~~ Proper subset ( $\subset$ )

when  $A \subseteq B$  but  $A \neq B$

Symmetric - प्रतिव्यक्त / उल्टा