Quantification Theory

Singular Proposition :

There are certain arguments whose validity cannot be proved by the methods used before. Let us take the ~~ex~~ example of a valid argument :

All humans are mortal
Socrates ~~is~~ is Human
Socrates ~~is~~ is Mortal.

In this argument none of the proposition is compound. So no methods used so far are applicable here.
The simplest prop. is the 2nd premise of the argument.

Socrates is ~~a~~ human — is a singular prop. An affirmative particular prop. Here 'Socrates' is subject & 'human' is the pred. The subject refers to a definite individual predi. refers to an attribute belonging to that individual.

Different singular props may have the same subject. e.g. S is mortal

S is female, S is active, S is beautiful

1, 3 – T, 2, 4 – F

Again Different singular props may have the same predicate e.g.

Aristotle is human, ~~Aris is Chicago,~~ Brazil is human, Calcutta is human, Descartes is human:

1, 4 – T, 2, 3, – F.

In order to express an individual we shall use a small letter as a symbolic, these symbols are called individual constants. It is convenient to use the initial letter of its name.

For example – Socrates – s

Aristotle – a as symbols.

On the other hand, as symbols of the predicates we shall use capital letters, e.g. ~~being~~ being human, being mortal, being active – we shall use H, M, W

For symbolising a singular prop. we shall use first the symbol of the attribute and then of the individual e.g. 'Socrates is human' – its symbolic expression is Hb, Aristotle is human' – Ha, Ram is human – Hr

For expressing the attribute 'being human' in respect of any individual we can use the symbol – Hx or H(x), ~~Here~~ Here ~~x~~ is individual variable. i.e. place marker – where by writing the symbols, a, b, c. etc. we can form singular ~~props~~ propositions. The singular props – Ha, Hb, Hc – are either ~~false~~ true or false. But Hx is neither true nor false, because Hx is not a proposition; it is only propositional function. Any singular prop. is a substitution instance of a propositional function.

Quantification :

By generalization or quantification we can get proposition.

Besides singular propositions, in other propositions also the predicate occurs e.g. 'Everything is mortal' and something is ~~mortal~~ beautiful – in these two propositions nothing is said about any individual; these two propositions are <u>general propositions</u>.

Given any x, x is mortal
or given any x, Mx

Traditional Subject – Predicate propositions:

Four kinds of Traditional props. are given below:

A Univ. Affirmative $(x)(Hx \supset Mx)$
E Univ. Negative $(x)(Hx \supset \sim Px)$
I Particular Affirmative $(\exists x)(Fx \cdot Wx)$
O Particular Negative ~~(∃x) (Fx · ~Wx)~~ $(\exists x)(Fx \cdot \sim Wx)$

First let us take A – All humans are mortal

Given any x, if x is human then x is mortal.

Now, if we use signs of propositional functions and quantifiers, we can express A proposition thus: $(x)[Hx \supset Mx)$

The substitution instances of the propositional function $Hx \supset Mx$ will be hypothetical propositions $Ha \supset Ma$, $Hb \supset Mb$ etc.

Let us take ~~A to~~ E – No humans are perfect.

Given any x, if x is human then x is not perfect.

i.e. $(x)(Hx \supset \sim Px)$

Let us take I – Some flowers are white

There is atleast one x such that x is ~~human~~ flower and x is white

~~at~~ i.e. $(\exists x)(Fx \cdot Wx)$

Let us take O – Some flowers are not white.

There is atleast one x such that x is flower and x is not white.

i.e. $(\exists x)(Fx \cdot \sim Wx)$

The Greek letters phi (ϕ) and psi (ψ) are used to represent any predicates whatever.

Rules of Inference: Quantification

Universal Instantiation U.I. $(x)(\phi x)$

$\therefore \phi v$

(where v is any individual symbol)

Universal Generalization U.G ϕy

$\therefore (x)(\phi x)$

(where y denotes 'any arbitrarily selected individual')

Existential Instantiation E.I. $(\exists x)(\phi x)$

$\therefore \phi v$

Existential Generalization E.G. ϕv

$\therefore (\exists x)(\phi x)$